Name: _____ Date: _____

UNIT 4

5.4.5 — Sine Rule

opposite pairs • missing sides and angles • ambiguous case • modelling

A. MATCH OPPOSITE SIDES AND ANGLES

Lowercase side letters are opposite the matching uppercase angle.

Side	Opposite angle	Complete the pair
a	A	a ↔ A
b	_____	b ↔ _____
c	_____	c ↔ _____

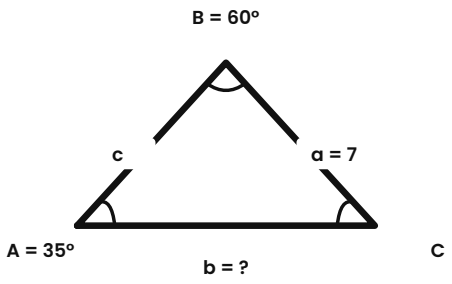
$$a/\sin A = b/\sin B = c/\sin C$$

Use a known opposite side-angle pair with the side or angle you need.

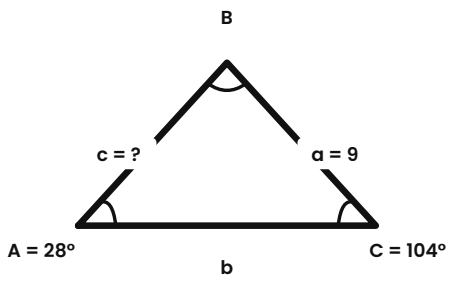
B. FIND MISSING SIDES

Give answers to 1 decimal place.

Diagrams are not to scale.



B1



B3

1. A = 35°, a = 7, B = 60°.
   b = _____
2. A = 52°, a = 11, B = 73°.
   b = _____
3. A = 28°, a = 9, C = 104°.
   c = _____
4. B = 47°, b = 12, C = 81°.
   c = _____

C. CHOOSE THE CORRECT SETUP

1. For A = 40°, a = 8 and B = 65°, circle the correct equation:
   $8/\sin 40^\circ = b/\sin 65^\circ$ or $8/\sin 65^\circ = b/\sin 40^\circ$
2. Explain why matching opposite pairs matters.

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UNIT 4

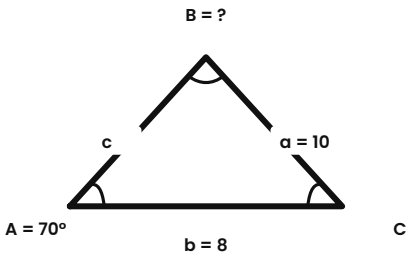
5.4.5 — Sine Rule

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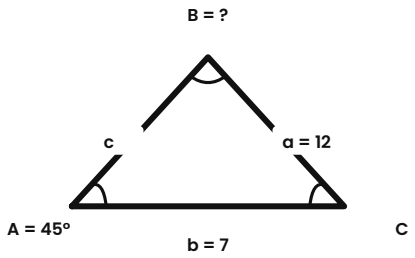
D. FIND MISSING ANGLES

Give angles to 1 decimal place and check whether the supplementary angle is possible.

Diagrams are not to scale.



D1



D2

1. A = 70°, a = 10, b = 8. Find B.

   B = _____

2. A = 45°, a = 12, b = 7. Find B.

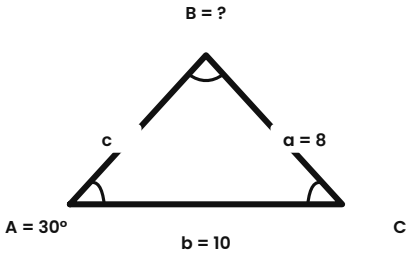
   B = _____

3. A = 50°, a = 12, b = 9. Explain why only one value of B gives a triangle.

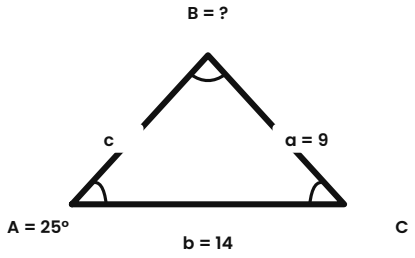
E. THE AMBIGUOUS CASE

For SSA data, inverse sine may give one acute angle, but its supplement can also have the same sine.

Diagrams are not to scale. In each SSA case, check the acute and obtuse possibilities for angle B.



E1



E2

1. A = 30°, a = 8, b = 10. Find **all** possible values of B.

   B = _____ or **B =** _____

2. A = 25°, a = 9, b = 14. Find **all** possible values of B.

   B = _____ or **B =** _____

3. For each problem, explain how the angle sum confirms whether both answers are valid.

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UNIT 4

5.4.5 — Sine Rule

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F. ONE TRIANGLE, TWO TRIANGLES OR NO TRIANGLE?

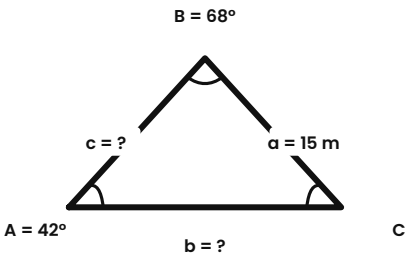
1. A = 35°, a = 7, b = 15. Predict the number of possible triangles and justify.

2. A = 50°, a = 12, b = 9. Predict the number of possible triangles and justify.

G. SOLVE A SURVEY TRIANGLE

A survey triangle has A = 42°, a = 15 m and B = 68°.

Diagram is not to scale.



G. Survey triangle

1. Find b.

   b = _____

2. Find C.

   C = _____

3. Find c.

   c = _____

4. Which side is longest? Explain using the angles.

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UNIT 4

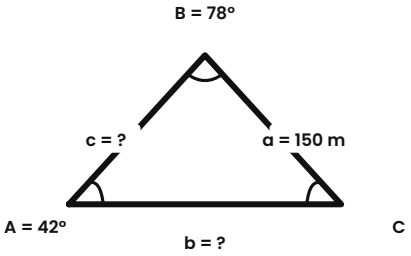
5.4.5 — Sine Rule

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H. LIGHTHOUSE OBSERVATIONS

Two observation points and a lighthouse form a triangle. Angle A = 42°, angle B = 78°, and side a = 150 m.

Diagram is not to scale.



H. Lighthouse triangle

1. Find side b.

   b = _____

2. Find angle C.

   C = _____

3. Find side c.

   c = _____

4. Interpret the longest side in the geometry of the observation triangle.

I. FIND AND CORRECT THE ERRORS

1. A student uses A = 35°, a = 7 and B = 60°, then writes $7/\sin 60^\circ = b/\sin 35^\circ$. Explain the error and give the correct setup.

2. A student finds B = 38.7° in an SSA problem and stops. What additional angle must be tested, and why?

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UNIT 4

5.4.5 — Sine Rule

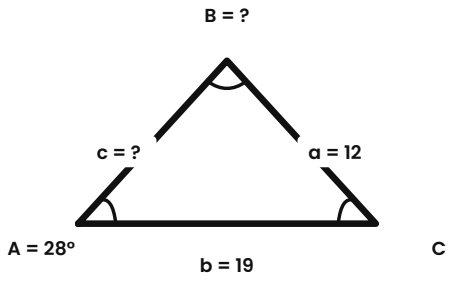
opposite pairs • missing sides and angles • ambiguous case • modelling

EXTENSION

J. TWO POSSIBLE TRIANGLES — COMPLETE BOTH

A = 28°, a = 12 and b = 19.

Diagram is not to scale. This SSA setup may produce two different triangles.



J. Two possible triangles

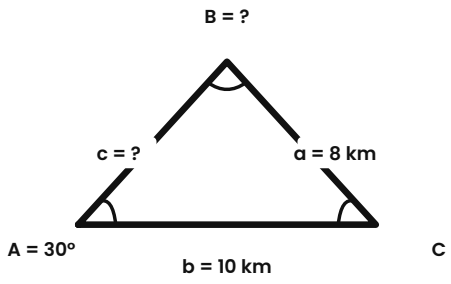
1. Find both possible values of B.
2. For each value of B, find C.
3. For each triangle, find c.

EXTENSION

K. CONTEXT SELECTS THE SOLUTION

A navigation triangle has A = 30°, a = 8 km and b = 10 km. The map shows that angle B is obtuse.

Diagram is not to scale. The map indicates that angle B is obtuse.



K. Navigation triangle

1. Find the two mathematical possibilities for B.
2. Select the value consistent with the map.
3. Find C and c for the selected triangle.

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UNIT 4

5.4.5 — Sine Rule

opposite pairs • missing sides and angles • ambiguous case • modelling

EXTENSION

L. IS THE DATA POSSIBLE?

1. A = 35°, a = 7 and b = 15. Use the sine rule to show why no triangle exists.

2. Explain what it means geometrically when the calculation gives $\sin B > 1$.

M. REASONING CHECK

A triangle has angles A = 25°, B = 60°, C = 95°. Without calculating any side lengths, order a, b and c from shortest to longest and explain.

ANSWER KEY

Name: _____ Date: _____

UNIT 4

5.4.5 — Sine Rule

opposite pairs • missing sides and angles • ambiguous case • modelling

A. OPPOSITE PAIRS AND THE RULE

b ↔ B and c ↔ C. The sine rule is $a/\sin A = b/\sin B = c/\sin C$.

B. MISSING SIDES

1. $b = 7\sin 60^\circ/\sin 35^\circ$ **≈ 10.6.**
2. $b = 11\sin 73^\circ/\sin 52^\circ$ **≈ 13.3.**
3. $c = 9\sin 104^\circ/\sin 28^\circ$ **≈ 18.6.**
4. $c = 12\sin 81^\circ/\sin 47^\circ$ **≈ 16.2.**

C. CORRECT SETUP

1. **Correct:** $8/\sin 40^\circ = b/\sin 65^\circ$.
2. **Each side must be paired with the angle directly opposite it; otherwise the ratios do not represent the same triangle relationship.**

D. MISSING ANGLES

1. $\sin B = 8\sin 70^\circ/10$; **B ≈ 48.7°. The supplement is 131.3°, but 70° + 131.3° > 180°, so it is invalid.**
2. $\sin B = 7\sin 45^\circ/12$; **B ≈ 24.4°. The supplement is 155.6°, but 45° + 155.6° > 180°.**
3. **B ≈ 35.1°. The supplement 144.9° is impossible because 50° + 144.9° > 180°.**

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UNIT 4

5.4.5 — Sine Rule

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E. THE AMBIGUOUS CASE

1. $\sin B = 10\sin 30^\circ / 8 = 0.625$. **B ≈ 38.7° or 141.3°. Both are valid because 30° + either value is less than 180°.**
2. $\sin B = 14\sin 25^\circ / 9$ **≈ 0.6574. B ≈ 41.1° or 138.9°. Both are valid because 25° + either value is less than 180°.**
3. **Check the supplement** $180^\circ - B$ **and then verify that A + B < 180°.**

F. ONE, TWO OR NO TRIANGLE?

1. $\sin B = 15\sin 35^\circ / 7$ **≈ 1.229. Since a sine value cannot exceed 1, no triangle exists.**
2. **The acute value is about 35.1°. Its supplement makes the angle sum exceed 180°, so there is one triangle.**

G. SURVEY TRIANGLE

1. $b = 15\sin 68^\circ / \sin 42^\circ$ **≈ 20.8 m.**
2. **C = 180° − 42° − 68° = 70°.**
3. $c = 15\sin 70^\circ / \sin 42^\circ$ **≈ 21.1 m.**
4. **c is longest because C = 70° is the largest angle.**

H. LIGHTHOUSE OBSERVATIONS

1. $b = 150\sin 78^\circ / \sin 42^\circ$ **≈ 219.3 m.**
2. **C = 180° − 42° − 78° = 60°.**
3. $c = 150\sin 60^\circ / \sin 42^\circ$ **≈ 194.1 m.**
4. **b is longest because it lies opposite the largest angle, B = 78°.**

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UNIT 4

5.4.5 — Sine Rule

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I. ERROR CORRECTION

1. **The angle-side pairs are reversed. Correct:** $7/\sin 35^\circ = b/\sin 60^\circ$.
2. **Also test the supplement:** $180^\circ - 38.7^\circ = 141.3^\circ$. **Sine has the same value for supplementary angles, so SSA can produce a second triangle.**

EXTENSION

J. TWO POSSIBLE TRIANGLES

$\sin B = 19\sin 28^\circ/12$, **so B ≈ 48.0° or 132.0°. Then C ≈ 104.0° or 20.0°. The corresponding third sides are** $c = 12\sin C/\sin 28^\circ$, **giving c ≈ 24.8 or 8.7.**

EXTENSION

K. CONTEXT SELECTS THE SOLUTION

The two possibilities are B ≈ 38.7° and 141.3°. The map requires an obtuse B, so choose 141.3°. Then C ≈ 8.7° and $c = 8\sin 8.7^\circ/\sin 30^\circ$ **≈ 2.4 km.**

EXTENSION

L. IS THE DATA POSSIBLE?

1. $\sin B = 15\sin 35^\circ/7$ **≈ 1.229, so no real angle B exists.**
2. **Geometrically, the supplied side lengths and angle cannot close to form a triangle.**

M. REASONING CHECK

Larger angles lie opposite longer sides. Since 25° < 60° < 95°, the sides satisfy a < b < c.