Name: _____ **Date:** _____

UNIT 2

5.2.2 — Quadratic Formula

Quadratic Formula • Coefficients • Discriminant • Exact & Approximate Roots • Context & Interpretation

A. IDENTIFY COEFFICIENTS AND READ THE DISCRIMINANT

First write the equation in standard form $ax^2 + bx + c = 0$. Then identify **a**, **b** and **c**.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Delta = b^2 - 4ac$$

1. For $x^2 + 6x + 5 = 0$, state a, b and c. Calculate Δ and state the number of real roots.
2. Repeat for $2x^2 - 3x - 2 = 0$.
3. Repeat for $x^2 + 4x + 4 = 0$.
4. Repeat for $3x^2 + 2x + 5 = 0$.

B. USE THE QUADRATIC FORMULA

Show the discriminant and keep the ± sign until both solutions have been found.

1. Solve $x^2 + 5x + 1 = 0$. Give the exact roots, then decimals to 2 d.p.
2. Solve $2x^2 - 3x - 2 = 0$.
3. Solve $3x^2 + 2x - 1 = 0$.

C. EXACT AND APPROXIMATE ANSWERS

State an exact surd form before rounding when the square root does not simplify to an integer.

1. Solve $2x^2 + 4x - 7 = 0$. Give exact solutions and decimals to 2 d.p.
2. Solve $x^2 - 4x - 1 = 0$. Give exact solutions and decimals to 2 d.p.
3. Explain why rounding should be left until the final step.

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D. USE THE DISCRIMINANT WITHOUT SOLVING

Use the sign of Δ to predict the number of real roots before applying the full formula.

1. State the number of real roots of $x^2 - 2x + 5 = 0$.
2. State the number of real roots of $4x^2 + 4x + 1 = 0$.
3. State the number of real roots of $5x^2 - x - 3 = 0$.
4. Find all values of k for which $x^2 + kx + 9 = 0$ has one repeated real root.
5. **Extended:** Find the values of k for which $2x^2 + kx + 8 = 0$ has no real roots.

E. FIND AND CORRECT THE ERROR

Identify the mathematical mistake, then write the corrected step.

1. For $2x^2 - 5x - 3 = 0$, a student writes $b = 5$. What should b be, and why?
2. A student writes $x = \frac{-b \pm \sqrt{\Delta}}{2}$. Correct the formula.
3. For $x^2 + 6x + 10 = 0$, a student obtains $\Delta = -4$ and continues to write two real decimal roots. Explain the error.
4. Why must the ± symbol not be replaced by only +?

F. QUADRATIC FORMULA IN CONTEXT

A mathematically valid root may still be outside the physical domain of a model.

1. A ball's height in metres is modelled by $h(t) = -5t^2 + 20t + 1$. Find when it reaches the ground. State both roots and identify the meaningful one for $t \geq 0$.
2. **Extended:** A flare is modelled by $h(t) = -4.9t^2 + 18t + 2.5$. Find when it reaches the ground to 2 d.p. and interpret the other root.

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UNIT 2

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G. EXTENDED DISCRIMINANT CHALLENGE

Use Δ as an equation or inequality when a coefficient is unknown.

1. Find all values of m for which $x^2 + (m-2)x + m = 0$ has one repeated real root.
2. Find the values of k for which $3x^2 + kx + 12 = 0$ has two distinct real roots.
3. Can two different quadratic equations have the same discriminant? Explain briefly.
4. Verify that $x^2 + x - 12 = 0$ has discriminant 49, then solve it.

H. EXTENDED IB-STYLE APPLICATION

Form the quadratic first, then use the formula and reject values that do not satisfy the context.

1. A rectangle has side lengths $x + 3$ metres and $x - 2$ metres and area 54 m². Show that $x^2 + x - 60 = 0$.
2. Solve the equation using the quadratic formula.
3. Hence find the rectangle's dimensions to 2 d.p. Explain why the other value of x is rejected.
4. What does the rejected root demonstrate about solving equations in context?

_____

_____

I. EXPLAIN AND JUSTIFY

Use mathematical reasoning; a numerical answer alone is not sufficient.

1. Why can calculating Δ before using the full quadratic formula save time?
2. Why is an exact surd often preferable to an early decimal approximation?
3. When is the quadratic formula more useful than factorisation?
4. In a real-world model, what can “no real roots” mean?
5. Describe one way to check a root after using the formula.

ANSWER KEY

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5.2.2 — Quadratic Formula

ANSWER KEY • Quadratic Formula • Discriminant • Exact & Approximate Roots • Context & Interpretation

A. IDENTIFY COEFFICIENTS AND READ THE DISCRIMINANT

1. $x^2 + 6x + 5 = 0$: **a = 1, b = 6, c = 5; Δ = 36 - 20 = 16, so there are two real roots.**
2. $2x^2 - 3x - 2 = 0$: **a = 2, b = -3, c = -2; Δ = 9 + 16 = 25, so there are two real roots.**
3. $x^2 + 4x + 4 = 0$: **a = 1, b = 4, c = 4; Δ = 16 - 16 = 0, so there is one repeated real root.**
4. $3x^2 + 2x + 5 = 0$: **a = 3, b = 2, c = 5; Δ = 4 - 60 = -56, so there are no real roots.**

B. USE THE QUADRATIC FORMULA

1. $x^2 + 5x + 1 = 0$: **Δ = 21;** $x = \frac{-5 \pm \sqrt{21}}{2}$**, so x ≈ -0.21 or -4.79.**
2. $2x^2 - 3x - 2 = 0$: **Δ = 25;** $x = \frac{3 \pm 5}{4}$**, so x = 2 or x = -1/2.**
3. $3x^2 + 2x - 1 = 0$: **Δ = 16;** $x = \frac{-2 \pm 4}{6}$**, so x = 1/3 or x = -1.**

C. EXACT AND APPROXIMATE ANSWERS

1. $2x^2 + 4x - 7 = 0$: **Δ = 72;** $x = -1 \pm \frac{3\sqrt{2}}{2}$**, giving x ≈ 1.12 or -3.12.**
2. $x^2 - 4x - 1 = 0$: **Δ = 20;** $x = 2 \pm \sqrt{5}$**, giving x ≈ 4.24 or -0.24.**
3. **Rounding only at the end preserves the exact value through the calculation and avoids avoidable rounding error.**

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D. USE THE DISCRIMINANT WITHOUT SOLVING

1. $x^2 - 2x + 5 = 0$: **Δ = 4 - 20 = -16; no real roots.**
2. $4x^2 + 4x + 1 = 0$: **Δ = 16 - 16 = 0; one repeated real root.**
3. $5x^2 - x - 3 = 0$: **Δ = 1 + 60 = 61; two distinct real roots.**
4. $x^2 + kx + 9 = 0$: **$k^2 - 36 = 0$, so $k = \pm 6$.**
5. $2x^2 + kx + 8 = 0$: **$k^2 - 64 < 0$, so $-8 < k < 8$.**

E. FIND AND CORRECT THE ERROR

1. **b = -5 because the x-term is -5x.**
2. **The denominator must be 2a:** $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
3. **Δ = -4 means the square root in the formula is negative, so the equation has no real roots.**
4. **± represents two cases, one using + and one using −; omitting one sign can lose a valid root.**

F. QUADRATIC FORMULA IN CONTEXT

1. $h(t) = -5t^2 + 20t + 1$: **Δ = 420 and** $t = 2 \pm \frac{\sqrt{105}}{5}$ **≈ -0.05 or 4.05. The meaningful root is t ≈ 4.05 s because t ≥ 0.**
2. $h(t) = -4.9t^2 + 18t + 2.5$: **t ≈ -0.13 or 3.81, so the flare reaches the ground at about 3.81 s. The negative root represents a modelled zero before the chosen starting time and is outside the physical domain.**

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G. EXTENDED DISCRIMINANT CHALLENGE

1. $x^2 + (m-2)x + m = 0$: **For a repeated root, Δ = 0: (m − 2)² − 4m = 0, so m² − 8m + 4 = 0 and $m = 4 \pm 2\sqrt{3}$.**
2. $3x^2 + kx + 12 = 0$: **Two distinct real roots require k² − 144 > 0, so k < −12 or k > 12.**
3. **Yes. The discriminant depends on the combination b² − 4ac, so different coefficients can produce the same value.**
4. $x^2 + x - 12 = 0$: **Δ = 1 + 48 = 49. Thus $x = \frac{-1 \pm 7}{2}$, so x = 3 or x = −4.**

H. EXTENDED IB-STYLE APPLICATION

1. **(x + 3)(x − 2) = 54 ⇒ x² + x − 6 = 54 ⇒ x² + x − 60 = 0.**
2. **Δ = 241, so $x = \frac{-1 \pm \sqrt{241}}{2}$ ≈ 7.26 or −8.26.**
3. **x ≈ 7.26 gives dimensions 10.26 m by 5.26 m. The negative value makes x − 2 negative, so it cannot represent a side length.**
4. **An algebraic root must still be tested against the constraints and meaning of the original situation.**

I. EXPLAIN AND JUSTIFY

1. **Δ immediately tells whether there are two, one or no real roots, so unnecessary solving can sometimes be avoided.**
2. **An exact surd preserves the exact mathematical value and can be rounded later to the precision required.**
3. **The quadratic formula works for every quadratic equation with a ≠ 0, including equations that do not factorise neatly.**
4. **It can mean the model never reaches the required value within the real-number setting; for example, a height curve may never meet a given level.**
5. **Substitute the root back into the original equation and check that the left side is 0 (or sufficiently close when using rounded values).**