Name: _____ **Date:** _____

UNIT 1

5.1.3 — Percentage Error

Absolute Error • Exact and Approximate Values • Percentage Error • Accuracy • Reliability • Tolerance • Interpretation

A. CALCULATE PERCENTAGE ERROR

Use the exact or accepted value in the denominator. Show the absolute error first.

$$\text{percentage error} = \frac{|\text{approximate} - \text{exact}|}{\text{exact}} \times 100\%$$

1. An estimate is 72 and the exact value is 75. Find the percentage error.
2. A length is measured as 2.45 m; the accepted value is 2.50 m. Find the percentage error.
3. A model predicts 480 people; the actual count is 500. Find the percentage error.
4. A sensor reports 9.6 cm when the exact length is 10.0 cm. Find the absolute error and percentage error.

B. INTERPRET THE RESULT

Percentage error measures the size of the error relative to the exact value.

1. A temperature sensor reads 19.6°C when the calibrated value is 20.0°C. State the percentage error and whether the reading is above or below the exact value.
2. An estimate is 102 when the exact value is 100. Explain what a 2% error means in this context.
3. Why is a smaller percentage error usually interpreted as greater accuracy?
4. Explain why the absolute value is used in the numerator of the percentage-error formula.

C. COMPARE ACCURACY ACROSS DIFFERENT SCALES

Three instruments measure quantities on different scales. Calculate each percentage error and rank them from most to least accurate.

Instrument	Approximate value	Exact value	Percentage error
A	198 cm	200 cm	
B	49.2 kg	50.0 kg	
C	0.992 L	1.000 L	

1. Complete the table.
2. Rank A, B and C from most accurate to least accurate.
3. Why would comparing only the raw absolute errors be misleading?

Name: _____ **Date:** _____

UNIT 1

5.1.3 — Percentage Error

Absolute Error • Exact and Approximate Values • Percentage Error • Accuracy • Reliability • Tolerance • Interpretation

D. SAME ABSOLUTE ERROR, DIFFERENT PERCENTAGE ERROR

Both measurements have an absolute error of 5 units.

1. Device X reports 495 when the exact value is 500. Find the percentage error.
2. Device Y reports 15 when the exact value is 20. Find the percentage error.
3. Which device is more accurate relative to the quantity being measured?
4. Explain how the same absolute error can represent very different levels of accuracy.

E. REVERSE PERCENTAGE-ERROR PROBLEMS

Use percentage error to determine how far an approximation may be from an exact value.

$$\text{maximum absolute error} = \frac{p}{100} \times \text{exact value}$$

1. The exact value is 240 and an approximation has percentage error 2.5%. Find the absolute error.
2. If the approximation in question 1 is below the exact value, find the approximation.
3. If the approximation could be above or below the exact value, state both possible values.
4. An exact value is 80. A measurement must have percentage error no more than 1.5%. Find the permitted interval for the measured value.

F. DECIDE WHETHER A SENSOR MEETS A TOLERANCE

A calibrated mass is exactly 50.0 g. A sensor is acceptable if its percentage error is at most 1.5%.

Reading	Absolute error (g)	Percentage error	Accept?
49.6 g			
50.7 g			
49.0 g			

1. Complete the table.
2. Which readings meet the 1.5% requirement?
3. Explain why an acceptance rule should state a numerical tolerance rather than simply say “close enough”.

Name: _____ **Date:** _____

UNIT 1

5.1.3 — Percentage Error

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G. PERCENTAGE ERROR IN MODELLING

Models and forecasts can be evaluated by comparing predictions with observed values.

1. A population model predicts 12,450 people; the observed population is 12,000. Find the percentage error.
2. A second model predicts 740 sales; the observed number is 720. Find the percentage error.
3. Which model is more accurate by percentage error?
4. Why is percentage error more useful than comparing the raw errors of 450 people and 20 sales?

H. FIND AND CORRECT THE ERRORS

Each student method contains an error or incomplete interpretation.

1. For estimate 95 and exact value 100, a student calculates 5 ÷ 95 × 100%. Identify and correct the error.
2. A student writes percentage error = (approximate - exact) ÷ exact × 100%, obtaining -4%. What should be changed?
3. A student finds 0.03 and reports the percentage error as 0.03%. Explain the error.
4. A student says two measurements with the same absolute error must be equally accurate. Give a counterexample.

I. EXTENDED IB-STYLE CHALLENGE: COMPARE TWO INSTRUMENTS

The accepted length of a calibration rod is 2.400 m. Instrument A reads 2.370 m and instrument B reads 2.440 m.

1. Find the percentage error of instrument A.
2. Find the percentage error of instrument B.
3. Which instrument is more accurate? Justify using percentage error.
4. A laboratory requires percentage error below 1.5%. State which instrument(s) meet the requirement.
5. After calibration, instrument A reads 2.390 m. Calculate its new percentage error and comment on the improvement.

J. EXTENDED: DESIGN AN ACCEPTABLE MEASUREMENT INTERVAL

A manufacturer fills packets with an exact target mass of 350 g. The measuring system must have percentage error at most 0.8%.

$$E\left(1-\frac{p}{100}\right) \le A \le E\left(1+\frac{p}{100}\right)$$

1. Find the maximum permitted absolute error in grams.
2. Find the full interval of measured values that meets the 0.8% requirement.
3. Decide whether readings of 347.4 g, 352.9 g and 350.8 g are acceptable. Show calculations.
4. A manager claims that any reading within 3 g of 350 g is acceptable. Evaluate this claim.
5. Explain one reason why the acceptable percentage error might depend on the context of the measurement.

ANSWER KEY

Name: _____ Date: _____

UNIT 1

5.1.3 — Percentage Error

Absolute Error • Exact and Approximate Values • Percentage Error • Accuracy • Reliability • Tolerance • Interpretation

A. CALCULATE PERCENTAGE ERROR

1. Absolute error = 3; percentage error = 3 ÷ 75 × 100% = **4%**.
2. Absolute error = 0.05 m; percentage error = 0.05 ÷ 2.50 × 100% = **2%**.
3. Absolute error = 20; percentage error = 20 ÷ 500 × 100% = **4%**.
4. Absolute error = **0.4 cm**; percentage error = 0.4 ÷ 10.0 × 100% = **4%**.

B. INTERPRET THE RESULT

1. Percentage error = 0.4 ÷ 20.0 × 100% = **2%**; the reading is **below** the exact value.
2. The estimate differs from the exact value by **2% of the exact value**; here it is 2 units too high.
3. A smaller percentage error means the error is smaller **relative to the scale of the exact value**.
4. The absolute value makes percentage error describe **magnitude of error**, regardless of whether the estimate is high or low.

C. COMPARE ACCURACY ACROSS DIFFERENT SCALES

1. A: 2 ÷ 200 × 100% = **1%**; B: 0.8 ÷ 50 × 100% = **1.6%**; C: 0.008 ÷ 1 × 100% = **0.8%**.
2. Most to least accurate: **C, A, B**.
3. Absolute errors use different units/scales; percentage error normalises the error relative to each exact value.

Name: _____ Date: _____

UNIT 1

5.1.3 — Percentage Error

Absolute Error • Exact and Approximate Values • Percentage Error • Accuracy • Reliability • Tolerance • Interpretation

D. SAME ABSOLUTE ERROR, DIFFERENT PERCENTAGE ERROR

1. 5 ÷ 500 × 100% = **1%**.
2. 5 ÷ 20 × 100% = **25%**.
3. **Device X** is more accurate relative to its exact value.
4. The same raw difference is a much smaller fraction of 500 than of 20.

E. REVERSE PERCENTAGE-ERROR PROBLEMS

1. 2.5% of 240 = 0.025 × 240 = **6**.
2. Below the exact value: 240 - 6 = **234**.
3. Possible approximations: **234 or 246**.
4. Maximum error = 0.015 × 80 = 1.2, so **78.8 ≤ measurement ≤ 81.2**.

F. DECIDE WHETHER A SENSOR MEETS A TOLERANCE

1. 49.6 g: error 0.4 g, **0.8%**, accept. 50.7 g: error 0.7 g, **1.4%**, accept. 49.0 g: error 1.0 g, **2.0%**, reject.
2. **49.6 g and 50.7 g** meet the requirement.
3. A numerical tolerance gives an objective, repeatable criterion and avoids subjective judgement.

Name: _____ Date: _____

UNIT 1

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G. PERCENTAGE ERROR IN MODELLING

1. 450 ÷ 12,000 × 100% = **3.75%**.
2. 20 ÷ 720 × 100% ≈ **2.78%**.
3. The second model is more accurate because **2.78% < 3.75%**.
4. Raw errors are measured on different scales; percentage error compares each error relative to its observed value.

H. FIND AND CORRECT THE ERRORS

1. Use the exact value as denominator: 5 ÷ 100 × 100% = **5%**.
2. Use the absolute value: |approximate - exact| ÷ exact × 100%; percentage error should be **non-negative**.
3. 0.03 is the decimal ratio; multiply by 100. The percentage error is **3%**.
4. Example: an error of 5 on 500 is 1%, while an error of 5 on 20 is 25%; equal absolute errors need not mean equal accuracy.

I. EXTENDED IB-STYLE CHALLENGE: COMPARE TWO INSTRUMENTS

1. A: |2.370−2.400| ÷ 2.400 × 100% = **1.25%**.
2. B: |2.440−2.400| ÷ 2.400 × 100% ≈ **1.67%**.
3. **Instrument A** is more accurate because it has the smaller percentage error.
4. Requirement is below 1.5%: **A passes; B fails**.
5. New A error: 0.010 ÷ 2.400 × 100% ≈ **0.417%**; calibration substantially improves its accuracy.

J. EXTENDED: DESIGN AN ACCEPTABLE MEASUREMENT INTERVAL

1. Maximum absolute error = 0.008 × 350 = **2.8 g**.
2. Acceptable interval: **347.2 g ≤ measurement ≤ 352.8 g**.
3. 347.4 g: 2.6/350×100≈**0.743%**, accept. 352.9 g: 2.9/350×100≈**0.829%**, reject. 350.8 g: 0.8/350×100≈**0.229%**, accept.
4. The claim is **false**: 3 g corresponds to about 0.857%, which exceeds 0.8%.
5. Acceptable error depends on consequences, safety, precision needs, cost, and the purpose of the measurement.