Name: _____ Date: _____

UNIT 2

4.2.3 - Factorising Quadratics

Meaning • Inverse of expansion • Product and sum

A. WHAT FACTORISING MEANS

Factorising rewrites a sum as a product. Expansion and factorisation are inverse processes.

$x^2 + 5x + 6 = (x + 2)(x + 3)$

1. What two linear factors multiply to give the quadratic shown?
2. What is the constant term c?
3. What is the coefficient b of x?
4. Explain why expanding the factors checks the factorisation.

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B. PRODUCT AND SUM RELATIONSHIP

For $x^2 + bx + c$: product = c, sum = b

1. For $x^2 + 7x + 12$, what must the two numbers multiply to?
2. What must the same two numbers add to?
3. List the positive factor pairs of 12.
4. Which pair works, and why?

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C. READ B AND C

For each quadratic, state b and c only.

1. $x^2 + 9x + 20$
2. $x^2 - 5x + 6$
3. $x^2 + x - 12$
4. $x^2 - 2x - 15$
5. $x^2 - 10x + 21$

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Name: _____ Date: _____

UNIT 2

4.2.3 - Factorising Quadratics

Reliable method • Factor pairs • Verification

D. RELIABLE FACTORISING METHOD

Put the source lesson steps in order:

check the sum • write and verify • find factor pairs • read the coefficients

1. Write the four steps in order.

2. Why are both the product and the sum needed?

3. Why should the final brackets be expanded to check?

E. WORKED EXAMPLE

$x^2 + 8x + 15 = (x + 3)(x + 5)$

1. List the factor pairs of 15.
2. Which pair adds to 8?
3. Write the factorisation.
4. Expand the factorisation to verify it.

F. FACTORISE: POSITIVE B AND POSITIVE C

1. $x^2 + 9x + 20$
2. $x^2 + 11x + 24$
3. $x^2 + 10x + 21$
4. $x^2 + 13x + 40$
5. $x^2 + 15x + 56$

Name: _____ Date: _____

UNIT 2

4.2.3 - Factorising Quadratics

Sign patterns • Positive constant • Negative constant

G. READ THE SIGN PATTERN

$x^2 + 6x + 8 = (x + 2)(x + 4)$

$x^2 + x - 6 = (x + 3)(x - 2)$

$x^2 - 7x + 10 = (x - 5)(x - 2)$

1. If c is positive, what can you say about the signs of the two factors?
2. If c is negative, what can you say about the signs?
3. If b is negative and c is positive, which equal sign pattern is needed?
4. Why must the value of b still be checked even after deciding the sign pattern?

H. FACTORISE: BOTH FACTORS NEGATIVE

1. $x^2 - 5x + 6$
2. $x^2 - 9x + 20$
3. $x^2 - 11x + 24$
4. $x^2 - 13x + 36$

I. FACTORISE: OPPOSITE SIGNS

1. $x^2 + x - 12$
2. $x^2 - 2x - 15$
3. $x^2 - 4x - 12$
4. $x^2 + 2x - 24$
5. $x^2 - 7x - 18$

Name: _____ Date: _____

UNIT 2

4.2.3 - Factorising Quadratics

Negative constant • Worked method • Sign reasoning

J. WORKED EXAMPLE WITH NEGATIVE C

$x^2 + 2x - 15 = (x + 5)(x - 3)$

1. Why must the factor signs be opposite?
2. List integer factor pairs of -15.
3. Which pair has sum 2?
4. Expand the answer to verify it.

K. FIND THE PAIR BEFORE FACTORISING

For each quadratic, first write the required **product** and **sum**, then factorise.

1. $x^2 + 4x - 21$
2. $x^2 - 3x - 28$
3. $x^2 + 5x - 36$
4. $x^2 - 8x + 15$

L. EXPLAIN THE SIGNS

1. Why does $x^2 - 8x + 15$ have two negative factors?
2. Why does $x^2 + 5x - 36$ have opposite signs?
3. Why is the larger-magnitude factor positive in $x^2 + 5x - 36$?

Name: _____ Date: _____

UNIT 2

4.2.3 - Factorising Quadratics

Error correction • Special cases • Integer factor checks

M. ERROR CORRECTION

A student says: "$x^2 + 7x + 12 = (x + 2)(x + 6)$ because $2 \times 6 = 12$."

$x^2 + 7x + 12 = (x + 3)(x + 4)$

1. Which condition has the student ignored?
2. What is $2 + 6$?
3. Find the correct factor pair.
4. Explain why checking only the product is insufficient.

N. A ZERO MIDDLE COEFFICIENT

$x^2 - 25 = (x + 5)(x - 5)$

1. What must the two factor numbers add to?
2. Why must they be opposites?
3. Factorise $x^2 - 49$.
4. Factorise $x^2 - 81$.

O. DOES IT FACTORISE OVER THE INTEGERS?

For each expression, decide whether an integer factor pair exists. If it does, factorise.

1. $x^2 + 6x + 5$
2. $x^2 + 4x + 5$
3. $x^2 - x - 20$
4. $x^2 + 2x + 7$

Name: _____ Date: _____

UNIT 2

4.2.3 - Factorising Quadratics

Quadratics in context • Rectangle dimensions • Verification

P. RECTANGULAR AREA

A rectangular garden has area $x^2 + 9x + 20$ square metres.

$x^2 + 9x + 20 = (x + 4)(x + 5)$

1. Find two linear expressions that can represent its dimensions.
2. Explain how the product and sum relationships identify them.
3. If $x = 3$, calculate the area using the original quadratic.
4. Check the same area using the factorised dimensions.

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_____

Q. COURTYARD DIMENSIONS

A courtyard has area $x^2 + 11x + 30$ square metres.

$x^2 + 11x + 30 = (x + 5)(x + 6)$

1. Find the possible dimensions.
2. Find the area when $x = 4$ using the factors.
3. Explain why negative lengths would be rejected in this context.

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R. REVERSE AREA MODEL

A rectangle has dimensions $x + 7$ and $x - 2$.

1. Expand to find its quadratic area expression.
2. Use factorisation to recover the dimensions from your quadratic.
3. State a condition on x so that both side lengths are positive.

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Name: _____ Date: _____

UNIT 2

4.2.3 - Factorising Quadratics

General structure • Reverse coefficients • Extended reasoning

S. WHY THE METHOD WORKS

$(x + p)(x + q) = x^2 + (p + q)x + pq$

1. Which part of the expansion creates the constant term?
2. Which two terms combine to give the middle coefficient?
3. Use the formula to explain why p and q must multiply to c and add to b.

_____

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T. FIND THE MISSING COEFFICIENT

1. $x^2 + kx + 24 = (x + 3)(x + 8)$. Find k.
2. $x^2 + kx - 18 = (x - 6)(x + 3)$. Find k.
3. $x^2 - 11x + k = (x - 5)(x - 6)$. Find k.
4. $x^2 + 2x + k = (x + 6)(x - 4)$. Find k.

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U. REVERSE-ENGINEER THE FACTORS

1. A monic quadratic has constant 35 and middle coefficient 12. Factorise it.
2. A monic quadratic has constant -40 and middle coefficient 3. Factorise it.
3. A monic quadratic has constant 48 and middle coefficient -14. Factorise it.
4. Explain how the sign of c narrows the search before testing sums.

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Name: _____ Date: _____

UNIT 2

4.2.3 - Factorising Quadratics

Build your own example • Reasoning • Final Review

V. BUILD YOUR OWN FACTORISATION

Choose two nonzero integers p and q.

1. Write $(x + p)(x + q)$.
2. Expand it to create a quadratic.
3. Hide the factors and list factor pairs of the constant term.
4. Recover p and q using product and sum.
5. Explain which sign pattern required the most care.

W. ALWAYS, SOMETIMES OR NEVER?

"A quadratic with a positive constant has two positive factors."

1. Decide: always, sometimes, or never.
2. Compare $x^2 + 7x + 12$ with $x^2 - 7x + 12$.
3. Explain how the middle coefficient determines whether the equal signs are positive or negative.

X. FINAL REVIEW

1. What must the factor pair multiply to?
2. What must the factor pair add to?
3. How do the signs of b and c help choose factor signs?
4. How can expansion verify a factorisation?
5. Factorise $x^2 + 5x + 6$.

ANSWER KEY

Name: _____ Date: _____

UNIT 2

ANSWER KEY - 4.2.3 Factorising Quadratics

ANSWER KEY • Meaning • Product and sum • Positive factors

A–C. MEANING AND COEFFICIENTS

$x^2 + 5x + 6 = (x + 2)(x + 3)$

Factorising rewrites a quadratic as a product. For $x^2 + bx + c$, the factor numbers multiply to c and add to b.
C: (b,c) = (9,20), (-5,6), (1,-12), (-2,-15), (-10,21).

D–E. RELIABLE METHOD

Read coefficients → find factor pairs → check the sum → write and verify.
Both conditions are needed because several pairs can share a product, but only the correct pair also has the required sum.

$x^2 + 8x + 15 = (x + 3)(x + 5)$

F. POSITIVE B AND C

$x^2 + 9x + 20 = (x + 4)(x + 5)$.
$x^2 + 11x + 24 = (x + 3)(x + 8)$.
$x^2 + 10x + 21 = (x + 3)(x + 7)$.
$x^2 + 13x + 40 = (x + 5)(x + 8)$.
$x^2 + 15x + 56 = (x + 7)(x + 8)$.

Name: _____ Date: _____

UNIT 2

ANSWER KEY - 4.2.3 Factorising Quadratics

ANSWER KEY • Sign patterns • Negative constants

G. SIGN PATTERNS

$x^2 + 6x + 8 = (x + 2)(x + 4)$

$x^2 + x - 6 = (x + 3)(x - 2)$

$x^2 - 7x + 10 = (x - 5)(x - 2)$

Positive c means the two factor numbers have the same sign. Negative c means opposite signs. With positive c and negative b, both factor numbers are negative.

H. BOTH FACTORS NEGATIVE

(x - 2)(x - 3); (x - 4)(x - 5); (x - 3)(x - 8); (x - 4)(x - 9).

I. OPPOSITE SIGNS

(x + 4)(x - 3); (x - 5)(x + 3); (x - 6)(x + 2); (x + 6)(x - 4); (x - 9)(x + 2).

J–L. NEGATIVE CONSTANT REASONING

$x^2 + 2x - 15 = (x + 5)(x - 3)$

K: (x + 7)(x - 3); (x - 7)(x + 4); (x + 9)(x - 4); (x - 3)(x - 5).

For positive c, equal signs are required; b decides whether both are positive or both negative. For negative c, opposite signs are required; b determines which magnitude carries the positive sign.

Name: _____ Date: _____

UNIT 2

ANSWER KEY - 4.2.3 Factorising Quadratics

ANSWER KEY • Errors • Special cases • Context

M. ERROR CORRECTION

$x^2 + 7x + 12 = (x + 3)(x + 4)$

The student checked the product 12 but ignored the sum. 2 + 6 = 8, not 7. The correct pair is 3 and 4.

N. ZERO MIDDLE COEFFICIENT

$x^2 - 25 = (x + 5)(x - 5)$

The factor numbers must add to 0, so they are opposites. $x^2 - 49 = (x + 7)(x - 7)$; $x^2 - 81 = (x + 9)(x - 9)$.

O. INTEGER FACTOR CHECKS

$x^2 + 6x + 5 = (x + 1)(x + 5)$.

$x^2 + 4x + 5$ does not have an integer factor pair with product 5 and sum 4.

$x^2 - x - 20 = (x - 5)(x + 4)$.

$x^2 + 2x + 7$ does not have an integer factor pair with product 7 and sum 2.

P–R. CONTEXT

$x^2 + 9x + 20 = (x + 4)(x + 5)$

At x = 3, area = 56 m² in both forms.

$x^2 + 11x + 30 = (x + 5)(x + 6)$

At x = 4, area = 90 m². Negative side lengths are not meaningful for physical dimensions.

$(x + 7)(x - 2) = x^2 + 5x - 14$. Both sides are positive when $x > 2$.

Name: _____ Date: _____

UNIT 2

ANSWER KEY - 4.2.3 Factorising Quadratics

ANSWER KEY • General structure • Reverse reasoning • Final Review

S. WHY THE METHOD WORKS

$(x + p)(x + q) = x^2 + (p + q)x + pq$

The constant comes from pq. The middle terms are px and qx, which combine to (p + q)x. Therefore pq = c and p + q = b.

T. MISSING COEFFICIENTS

1) k = 11
2) k = -3
3) k = 30
4) k = -24

U. REVERSE-ENGINEER

Constant 35, sum 12 → (x + 5)(x + 7).
Constant -40, sum 3 → (x + 8)(x - 5).
Constant 48, sum -14 → (x - 6)(x - 8).

V–W. BUILD AND REASON

V answers vary. Check that the chosen p and q reproduce the constant product and middle-term sum when expanded.
Sometimes. $x^2 + 7x + 12 = (x + 3)(x + 4)$, while $x^2 - 7x + 12 = (x - 3)(x - 4)$.

X. FINAL REVIEW

The pair multiplies to c and adds to b. The signs of c determine equal/opposite signs; b determines the resulting sum. Expansion verifies the original quadratic.

$x^2 + 5x + 6 = (x + 2)(x + 3)$

SUMMARY

Read b and c → list factor pairs of c → choose the pair whose sum is b → write the brackets → expand to verify.