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UNIT 4

3.4.5. The Pythagorean Theorem

Hypotenuse • Missing sides • Real-life modelling • Coordinate & 3D extensions

A. CORE IDEAS

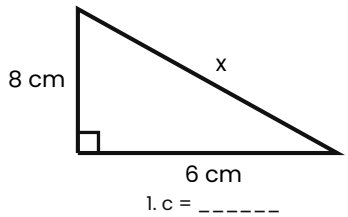
Complete the statements using the lesson terminology.

$a^2 + b^2 = c^2$ for a right-angled triangle.

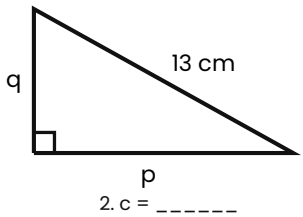
1. The side opposite the 90° angle is the _____ .
2. It is always the _____ side of a right triangle.
3. The theorem compares the _____ of the three side lengths.
4. Pythagoras' theorem should only be used when the triangle is _____ .

B. IDENTIFY THE HYPOTENUSE

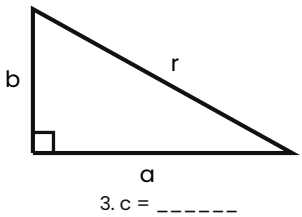
In each diagram, circle or state which labelled side must be c.



1. c = _____



2. c = _____



3. c = _____

Do not choose c because of where a side is drawn. Locate the right angle first.

C. FIND THE HYPOTENUSE

Use the theorem and show substitution. Give exact values when they are simple radicals.

1. Legs 6 cm and 8 cm.

2. Legs 7 m and 24 m.

3. Legs 9 cm and 12 cm.

4. Legs 10 cm and 10 cm.

5. Legs 5.4 m and 7.2 m.

6. Legs 1.2 m and 3.5 m.

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D. FIND A SHORTER SIDE

$a = \sqrt{c^2 - b^2}$ Subtract when the hypotenuse is known.

1. c = 13 cm, one leg = 5 cm.

2. c = 17 m, one leg = 8 m.

3. c = 25 cm, one leg = 15 cm.

4. c = 10 m, one leg = 6 m.

5. c = 7.3 m, one leg = 4.8 m. Give 3 s.f.

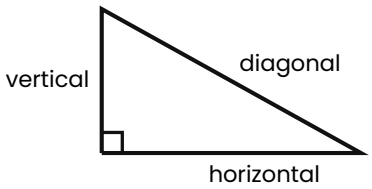
6. c = 20 cm, one leg = 11 cm. Give 3 s.f.

E. EXACT OR ROUNDED?

1. A square has side 8 cm. Find its diagonal in exact form and to 3 s.f.
2. A 7 cm by 11 cm rectangle has diagonal d. Find d to 3 s.f.
3. Explain one advantage of keeping an exact radical until the final step.

F. REAL MEASUREMENTS

1. A 15 m ladder reaches 12 m up a wall. How far is its foot from the wall?
2. A ramp is 5 m long and rises 3 m. Find its horizontal run.
3. A rectangular park is 60 m by 80 m. Find the corner-to-corner diagonal.



Sketch the right triangle before calculating.

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G. DOES THE TRIANGLE HAVE A RIGHT ANGLE?

Extended reasoning: use the longest side as c and compare c^2 with the sum of the other two squares.

Sides	Calculation	Right-angled?
8, 15, 17		
7, 8, 11		
10, 24, 26		
9, 12, 16		

H. COORDINATE DISTANCE

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ This is Pythagoras applied to horizontal and vertical changes.

1. Find the distance between A(1,2) and B(7,10).

_____

2. Find the diagonal distance from P(-2,3) to Q(4,-5). Give an exact value and a decimal.

_____

3. A rectangle has vertices (1,1), (9,1), (9,7), (1,7). Find a diagonal and explain why both diagonals have the same length.

_____

_____

I. ERROR CORRECTION

1. A student knows c=13 and a=5 and writes $b^2=13^2+5^2$. Correct the method.
2. A student gets $c^2=169$ and reports c=169 cm. What step is missing?
3. A triangle has sides 4, 5 and 8. A student uses $4^2+5^2=8^2$ without checking. What should they conclude?
4. A drawing mixes 1.2 m and 80 cm. What must be done before applying the theorem?

_____

_____

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J. EXTENDED MODELLING: DIAGONAL ROUTES

A rectangular park is 60 m by 80 m. A walker can follow two sides or take a straight diagonal path.

1. Find the diagonal path length.
2. Find the length of the two-side route from one corner to the opposite corner.
3. How many metres are saved by using the diagonal?
4. What percentage of the two-side route is saved? Give 1 d.p.
5. Explain why the diagonal is the shortest straight route between the corners.

_____

_____

K. CHALLENGE MODELLING

1. 55-inch 16:9 screen

The width and height are in the ratio 16:9 and the diagonal is 55 inches.

$(16k)^2 + (9k)^2 = 55^2$

Find k, then estimate the width and height to 1 d.p.

_____

_____

2. Cable across a room

A room is 9 m by 12 m with height 2.8 m. A cable runs from one floor corner to the opposite ceiling corner.

First find the floor diagonal d, then use $L = \sqrt{d^2 + h^2}$.

Find the cable length to 2 d.p.

_____

_____

L. ALWAYS, SOMETIMES OR NEVER?

1. The longest side of every triangle is called the hypotenuse.
2. If $a^2+b^2=c^2$ and c is the longest side, the triangle is right-angled.
3. If both shorter sides are multiplied by 3, the hypotenuse is also multiplied by 3.
4. A right triangle can have two equal shorter sides.
5. If c^2 is greater than a^2+b^2, the triangle is right-angled.

Give a reason or counterexample for each.

ANSWER KEY

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UNIT 4

ANSWER KEY – 3.4.5

Hypotenuse • Missing sides • Real-life modelling • Coordinate & 3D extensions

A. CORE IDEAS

1. **hypotenuse**
2. **longest**
3. **squares**
4. **right-angled**

B. IDENTIFY THE HYPOTENUSE

1. **x** 2. **13 cm** 3. **r**. In each diagram it is opposite the right angle.

C. FIND THE HYPOTENUSE

1. $\sqrt{(6^2+8^2)}=$**10 cm**
2. $\sqrt{(7^2+24^2)}=$**25 m**
3. $\sqrt{(9^2+12^2)}=$**15 cm**
4. $\sqrt{200}=$**$10\sqrt{2}$ cm ≈14.1 cm**
5. $\sqrt{(5.4^2+7.2^2)}=$**9 m**
6. $\sqrt{(1.2^2+3.5^2)}=\sqrt{13.69}=$**3.70 m**

D. FIND A SHORTER SIDE

1. $\sqrt{(13^2-5^2)}=$**12 cm**
2. $\sqrt{(17^2-8^2)}=$**15 m**
3. $\sqrt{(25^2-15^2)}=$**20 cm**
4. $\sqrt{(10^2-6^2)}=$**8 m**
5. $\sqrt{(7.3^2-4.8^2)}=\sqrt{30.25}=$**5.50 m**
6. $\sqrt{(20^2-11^2)}=\sqrt{279}=$**16.7 cm**

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ANSWER KEY - 3.4.5

Hypotenuse • Missing sides • Real-life modelling • Coordinate & 3D extensions

E. EXACT OR ROUNDED?

1. $d=\sqrt{(8^2+8^2)}=$ **$8\sqrt{2}$ cm ≈11.3 cm**.
2. $d=\sqrt{170}=$ **13.0 cm** to 3 s.f.
3. Exact form avoids **premature rounding error** and preserves full accuracy for later calculations.

F. REAL MEASUREMENTS

1. $\sqrt{(15^2-12^2)}=$ **9 m**.
2. $\sqrt{(5^2-3^2)}=$ **4 m**.
3. $\sqrt{(60^2+80^2)}=$ **100 m**.

G. DOES THE TRIANGLE HAVE A RIGHT ANGLE?

8,15,17: $64+225=289=17^2$ → **yes**.

7,8,11: $49+64=113\neq121$ → **no**.

10,24,26: $100+576=676=26^2$ → **yes**.

9,12,16: $81+144=225\neq256$ → **no**.

H. COORDINATE DISTANCE

1. $\Delta x=6$, $\Delta y=8$, so $d=\sqrt{100}=$ **10**.
2. $\Delta x=6$, $\Delta y=-8$, so $d=\sqrt{100}=$ **10** exactly and 10.0 decimal.
3. Width 8, height 6; diagonal $=\sqrt{(8^2+6^2)}=$ **10**. Both diagonals span the same horizontal and vertical changes.

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UNIT 4

ANSWER KEY – 3.4.5

Hypotenuse • Missing sides • Real-life modelling • Coordinate & 3D extensions

I. ERROR CORRECTION

1. Subtract because 13 is the hypotenuse: $b^2=13^2-5^2=144$, so **b=12**.
2. Take the square root: **c=13 cm**.
3. $4^2+5^2=41$ while $8^2=64$, so **the triangle is not right-angled**.
4. Convert to one unit first, e.g. **1.2 m=120 cm**.

J. DIAGONAL ROUTES

1. Diagonal=**100 m**.
2. Two-side route=60+80=**140 m**.
3. Saving=**40 m**.
4. 40/140×100=**28.6%**.
5. The straight-line segment between two points is the shortest path; here it is the rectangle's diagonal.

K. CHALLENGE MODELLING

1. $337k^2=3025$, so $k\approx2.996$. Width=16k≈**47.9 in**; height=9k≈**27.0 in**.
2. Floor diagonal $d=\sqrt{(9^2+12^2)}=$**15 m**. Cable $L=\sqrt{(15^2+2.8^2)}=\sqrt{232.84}\approx$**15.26 m**.

L. ALWAYS, SOMETIMES OR NEVER?

1. **Sometimes.** "Hypotenuse" applies only to right triangles.
2. **Always.** This is the converse relationship with c the longest side.
3. **Always.** Scaling both legs by 3 scales c by 3.
4. **Sometimes.** An isosceles right triangle has equal legs.
5. **Never.** A right triangle requires equality, not $c^2>a^2+b^2$.