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UNIT 1

3.1.5 — Laws of Indices

IB Maths MYP • Year 3 • Unit 1 – Real Numbers, Indices and Accuracy
Multiply: Add • Divide: Subtract • Power of a Power: Multiply

A. KEY VOCABULARY: THE THREE INDEX LAWS

The laws never touch the base — they only combine the **exponents**, and only when the bases already match.

three laws — and one condition they all share

multiply $a^m \times a^n =$

divide $a^m \div a^n =$

power of a power $(a^m)^n =$

all three laws need the SAME BASE

different bases? no law applies — work each power out separately

1. $a^m \times a^n =$ _____ in words: _____
2. $a^m \div a^n =$ _____ in words: _____
3. $(a^m)^n =$ _____ in words: _____
4. **What condition must hold before any of the three laws can be used?**

   _____
5. **Which law carries the extra condition $a \neq 0$, and why?**

   _____

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3.1.5 — Laws of Indices

Which Law Applies?

B. WARM-UP: WHICH LAW APPLIES?

Write **add**, **subtract**, **multiply** or **no law**, then simplify.

Expression	Which law?	Simplified
$m^4 \times m^7$		
$p^{12} \div p^5$		
$(q^4)^3$		
$x^3 \times y^4$		
$2^5 \times 2^3$		
$3^4 \div 3^4$		

1. Why can nothing be done to $x^3 \times y^4$? _____

2. $2^5 \times 2^3$ can be written as a power and as a number: _____ **=** _____

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3.1.5 — Laws of Indices

Proof by Expanding

C. PROVING THE LAWS BY EXPANDING

Every law is just bookkeeping on the number of factors. Write each expansion out in full and the rule appears on its own.

expand, and the laws stop being rules to memorise

$x^3 \times x^4$ = (x·x·x)(x·x·x·x) = x^7

3 factors then 4 more — so 7 altogether: ADD

$y^5 \div y^2$ = $\frac{y \cdot y \cdot y \cdot y \cdot y}{y \cdot y}$ = y^3

2 factors cancel from 5 — so 3 are left: SUBTRACT

$(c^2)^3$ = $c^2 \cdot c^2 \cdot c^2$ = (c·c)(c·c)(c·c) = c^6

3 copies of 2 factors — so 6: MULTIPLY

---

the base never changes — only the count of factors does

1. $a^3 \times a^2$ = (_____)(_____) = _____ so we _____ the exponents

2. $b^5 \div b^2$: cancel _____ factors from _____ , leaving _____

3. $(c^2)^3$ = _____ = _____

4. Use expansion to explain why $(p^2)^5 = p^{10}$ and not p^7 :

_____

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3.1.5 — Laws of Indices

Worked Frames • Independent Practice

D. WORKED FRAMES

Identify the operation, check the bases match, then choose the law.

1. $a^6 \times a^2$ = _____
2. $b^9 \div b^4$ = _____
3. $(c^3)^4$ = _____
4. $2^5 \times 2^3$ = _____ = _____
5. $x^7 \div x^7$ = _____ = _____
6. $(2^2)^3$ = _____ = _____
7. **Write one of the above in expanded form to justify your answer:**

   _____

E. INDEPENDENT PRACTICE: CHOOSE THE CORRECT LAW

Check: never add or subtract exponents across different bases.

1. **Simplify** $m^4 \times m^7$ = _____
2. **Simplify** $p^{12} \div p^5$ = _____
3. **Simplify** $(q^4)^3$ = _____
4. **Evaluate** 5^0 = _____
5. **Write** 3^{-2} **as a fraction:** _____
6. **Simplify** $(2^3)^4 \div 2^5$ = _____ ÷ _____ = _____

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3.1.5 — Laws of Indices

Mixed Practice • Error Analysis

F. MIXED PRACTICE

Use brackets and exponents carefully to show the structure of each expression.

1. **Simplify** $r^{10} \div r^{6} \times r^{2}$ **=** _____
2. **Simplify** $(a^{2})^{3} \times a$ **=** _____
3. **Evaluate** 10^{-3} **as a decimal:** _____
4. **Simplify** $(x^{5}y^{3})^{2}$ **=** _____
5. **Explain why** 4^{-2} **is smaller than** 4^{0} **:**
   4^{-2} **=** _____ **and** 4^{0} **=** _____ **so**
   _____
6. **Is** $(m+n)^{2}$ **equal to** $m^{2}+n^{2}$ **? Test with** $m=n=1$ **:**
   $(m+n)^{2}$ **=** _____ $m^{2}+n^{2}$ **=** _____ **Verdict:** _____

G. FIND AND CORRECT THE ERROR

Each student has broken one law. Say what went wrong, then correct it.

1. **Ada: "** $x^{3} \times x^{5} = x^{15}$ **because** $3 \times 5 = 15$ **."**
   Error: _____ **Correction:** _____
2. **Ben: "** $x^{5} \div x^{5} = 0$ **because everything cancels."**
   Error: _____ **Correction:** _____
3. **Cara: "** $(x^{3})^{4} = x^{7}$ **."**
   Error: _____ **Correction:** _____
4. **Dev: "** $2^{3} \times 3^{2} = 6^{5}$ **."**
   Error: _____ **Correct value:** _____
5. **Eze: "** $3^{-2} = -9$ **."**
   Error: _____ **Correction:** _____

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UNIT 1

3.1.5 — Laws of Indices

Zero and Negative Indices

H. ZERO AND NEGATIVE INDICES

Each division by the base lowers the exponent by 1. Follow that pattern past a^1 and it tells you what a^0 and the negative indices must mean.

keep dividing by 3 and the pattern walks straight past zero

$3^3 = 27$
$3^2 = 9$ ↓ ÷3
$3^1 = 3$ ↓ ÷3
$3^0 = ?$ ↓ ÷3
$3^{-1} = ?$ ↓ ÷3
$3^{-2} = ?$ ↓ ÷3

continue the pattern past 3^1
what must 3^0 and 3^{-1} be?

1. 3^3 = _____ , 3^2 = _____ , 3^1 = _____ , 3^0 = _____

2. **Continue:** 3^{-1} = _____ , 3^{-2} = _____

3. 5^0 = _____ 2^{-3} = _____ 10^{-2} = _____

4. **State both rules:** a^0 = _____ **and** a^{-n} = _____ **, provided** _____

5. **Is** 3^{-2} **negative?** _____ **Explain what a negative index actually does:** _____

6. **Why is** 0^0 **not defined?** _____

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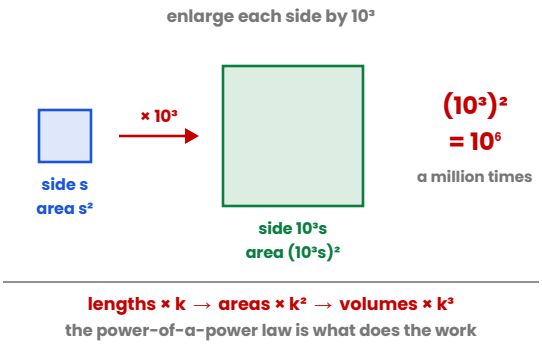
UNIT 1

3.1.5 — Laws of Indices

Scaling and Powers of Ten

I. REAL-WORLD CONNECTION: SCALING AND POWERS OF TEN

A square image is enlarged by a scale factor of 10^3 in each dimension.



1. Area scale factor = $(10^3)^2$ **=** _____ **in words:**

_____

2. If it were a solid enlarged the same way, the volume factor would be $(10^3)^3$ **=** _____

3. Use index laws: $(10^6 \times 10^3) \div 10^4$ **=** _____ ÷ _____ **=**

_____

4. Show the same calculation written out in full, and say which method you prefer:

_____

5. $10^8 \div 10^5$ **=** _____ $(10^2)^4$ **=** _____

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UNIT 1

3.1.5 — Laws of Indices

Inquiry

J. INQUIRY: ALWAYS, SOMETIMES OR NEVER?

Circle your answer, then state the condition precisely.

1. **"When two powers are multiplied, their exponents should be added."**
   Always / Sometimes / Never **Required condition:** _____
2. **"** $(a^m)^n = a^{mn}$ **."**
   Always / Sometimes / Never **because** _____
3. **"** $a^m \div a^n = a^{m-n}$ **."**
   Always / Sometimes / Never **because** _____
4. **"** $a^m + a^n = a^{m+n}$ **."**
   Always / Sometimes / Never **Try** $a = 2, m = n = 1$ **:** _____
5. **"** $(ab)^n = a^n b^n$ **."**
   Always / Sometimes / Never **because** _____

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UNIT 1

3.1.5 — Laws of Indices

Extra Practice • Negative Indices

K. EXTRA PRACTICE

1. **Simplify** $z^7 \times z^3$ **=** _____
2. **Simplify** $b^9 \div b^4$ **=** _____
3. **Evaluate** 7^0 **=** _____
4. **Write** 2^{-4} **as a fraction:** _____
5. **Simplify** $(t^3)^2 \times t^4$ **=** _____
6. **Which index law is easiest to justify using expanded form, and why?**

   _____

L. EXTENSION: NEGATIVE INDICES FROM THE QUOTIENT LAW

Negative indices are not a new invention — they are what the quotient law is **forced** to mean when the bottom exponent is larger than the top.

1. **Work out** $a^3 \div a^5$ **two ways.**

   By cancelling: $\frac{a \cdot a \cdot a}{a \cdot a \cdot a \cdot a \cdot a}$ **=** _____

   By the law: a^{3-5} **=** _____

   So a^{-2} **must equal** _____

2. 2^{-1} **=** _____ 2^{-2} **=** _____ 2^{-3} **=** _____ 2^{-4} **=** _____

3. 10^{-1} **=** _____ 10^{-2} **=** _____ 10^{-6} **=** _____

4. 5^{-1} **=** _____ 3^{-3} **=** _____ $\left(\frac{1}{2}\right)^{-1}$ **=** _____

   $\left(\frac{1}{2}\right)^{-2}$ **=** _____

5. **What does a negative index do to a fraction?** _____

6. **Put in order, smallest first:** $2^{-2},\ 2^0,\ 2^{-1},\ 2^2 \rightarrow$ _____

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UNIT 1

3.1.5 — Laws of Indices

Brackets

M. EXTENSION: BRACKETS OVER PRODUCTS AND QUOTIENTS

A bracket hands the power to **every** factor inside it — the numbers as well as the letters. This is the law behind Section F question 4.

a bracket shares the power with EVERY factor inside

$(ab)^n$

$(a/b)^n$

$(x^5y^3)^2$

$(2x)^3$ is $8x^3$, NOT $2x^3$ — the 2 is inside the bracket

but a bracket over a SUM shares nothing: $(a+b)^2$ is NOT $a^2 + b^2$

1. State the two laws: $(ab)^n$ = _____ $\left(\frac{a}{b}\right)^n$ = _____

2. $(2x)^3$ = _____ $(3y)^2$ = _____ $(5m)^2$ = _____

3. $(x^5y^3)^2$ = _____ $(2a^2b)^3$ = _____

4. $\left(\frac{x}{2}\right)^3$ = _____ $\left(\frac{2}{3}\right)^2$ = _____ $\left(\frac{a^2}{b}\right)^3$ = _____

5. A student writes $(2x)^3 = 2x^3$ **. Test it with** $x = 1$ **:** _____

What did they forget? _____

6. Does the same sharing work over a sum? Test $(a+b)^2$ **with** $a = 3, b = 4$ **:**

_____

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UNIT 1

3.1.5 — Laws of Indices

Longer Chains

N. EXTENSION: SIMPLIFYING LONGER CHAINS

With several operations, deal with any **brackets** first, then work left to right through the multiplications and divisions. Numbers and letters are handled separately.

1. $x^5 \times x^3 \div x^2$ = _____
2. $(m^3)^2 \times m^4 \div m^5$ = _____
3. $(2^3)^2 \div 2^4$ = _____ = _____
4. $6a^5 \div 2a^2$ = _____ $12x^7 \div 4x^3$ = _____
5. $(3x^2)^2 \times 2x^3$ = _____
6. $(2p^3)^3 \div 4p^5$ = _____ ÷ _____ = _____
7. **In question 6, what happened to the numbers and what happened to the letters?**

   _____

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UNIT 1

3.1.5 — Laws of Indices

Powers of Ten

O. EXTENSION: INDEX LAWS WITH POWERS OF TEN

The laws turn awkward multiplications of huge numbers into simple additions of exponents. Handle the leading numbers and the powers of ten separately.

1. $10^6 \times 10^3$ = _____ $10^9 \div 10^4$ = _____ $(10^2)^4$ = _____

2. $(3 \times 10^4) \times (2 \times 10^3)$:
   numbers: 3×2 = _____ **powers:** $10^4 \times 10^3$ = _____ **answer:** _____

3. $(8 \times 10^6) \div (2 \times 10^2)$ = _____

4. **The Earth has mass about** 6×10^{24} **kg and the Moon about** 7.3×10^{22} **kg.**
   Roughly how many times heavier is the Earth? _____ **(2 s.f.)**
   Which part of the calculation did the index law do for you? _____

5. **Write** $10^{-3} \times 10^5$ **as a single power:** _____ **and as an ordinary number:** _____

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UNIT 1

3.1.5 — Laws of Indices

What Is Not a Law

P. EXTENSION: WHAT IS *NOT* AN INDEX LAW

Four statements that look like laws. Each one is false, and a single counter-example is enough to prove it.

four things that look like laws and are not

- ✗ $(m + n)^2 = m^2 + n^2$
- ✗ $a^m + a^n = a^{m+n}$
- ✗ $2^3 \times 3^2 = 6^5$
- ✗ $3^{-2} = -9$

one counter-example is enough to kill a rule for ever

the laws only ever combine EXPONENTS, and only for the SAME BASE

1. $(m+n)^2 = m^2 + n^2$ **? Test** $m = 3,\ n = 4$ **:**
   LHS = _____ **RHS =** _____ **Verdict:** _____
2. $a^m + a^n = a^{m+n}$ **? Test** $a = 2,\ m = 3,\ n = 2$ **:**
   LHS = _____ **RHS =** _____ **Verdict:** _____
3. $2^3 \times 3^2 = 6^5$ **? LHS =** _____ **RHS =** _____ **Verdict:**
   _____
4. $3^{-2} = -9$ **? Correct value:** _____
5. **Each false rule breaks the same underlying condition. What is it?**
   _____

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UNIT 1

3.1.5 — Laws of Indices

Challenge

Q. CHALLENGE: PROVING AND USING THE LAWS

(1) Prove the product law from expanded form.

a^m **is** _____ **factors of** a **;** a^n **is** _____ **factors.**

Multiplying them puts _____ **factors in one product, which is** _____ **. Hence** $a^m \times a^n = a^{m+n}$ **.**

(2) Use the quotient law twice to force the other two rules.

$a^n \div a^n$ **equals** _____ **by cancelling, and** $a^{n-n} = a^0$ **by the law, so** a^0 **=**

_____

$a^0 \div a^n$ **equals** _____ **by the first result, and** a^{-n} **by the law, so** a^{-n} **=**

_____

(3) Solve for x **:**

$2^x = 32 \rightarrow$ _____ $3^x = 1 \rightarrow$ _____ $5^x = \frac{1}{25} \rightarrow$ _____

$2^x \times 2^3 = 2^7 \rightarrow$ _____ $(2^x)^3 = 2^{12} \rightarrow$ _____

(4) The laws also decide what a fractional index must mean.

If the power-of-a-power law holds, then $(a^{1/2})^2 = a^1$ **=** _____

So $a^{1/2}$ **must be** _____

Therefore $9^{1/2}$ **=** _____ **and** $8^{1/3}$ **=** _____

ANSWER KEY

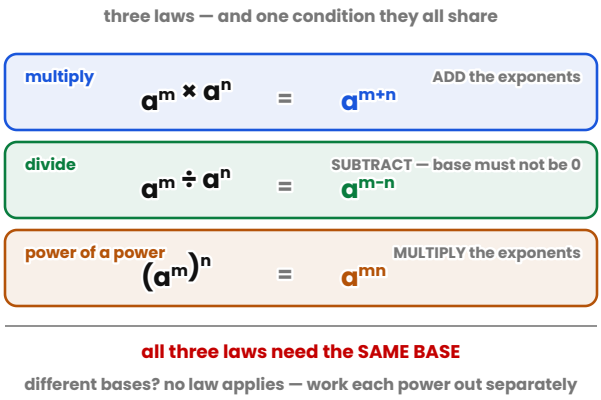
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UNIT 1

ANSWER KEY — 3.1.5 Laws of Indices

Teachers: the laws combine exponents, never the base

A. KEY VOCABULARY: THE THREE INDEX LAWS



1. $a^m \times a^n = a^{m+n}$ — same base, **add** the exponents.
2. $a^m \div a^n = a^{m-n}$ — same base, **subtract** the exponents.
3. $(a^m)^n = a^{mn}$ — **multiply** the exponents.
4. The **bases must be the same.** No law lets you combine exponents across different bases.
5. The **quotient** law, because dividing by a^n is impossible when $a = 0$ — you would be dividing by zero. (It is also why 0^0, which comes out of that law, is undefined.)

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UNIT 1

ANSWER KEY — 3.1.5 Laws of Indices

Teachers: different bases means no law at all

B. WARM-UP: WHICH LAW APPLIES?

Expression	Which law?	Simplified
$m^4 \times m^7$	**add**	m^{11}
$p^{12} \div p^5$	**subtract**	p^7
$(q^4)^3$	**multiply**	q^{12}
$x^3 \times y^4$	**no law**	x^3y^4
$2^5 \times 2^3$	**add**	$2^8 = 256$
$3^4 \div 3^4$	**subtract**	$3^0 = 1$

1. **The bases x and y are different, so there is no shared repeated factor to count. x^3y^4 is already as simple as it gets — writing it as xy^7 or $(xy)^7$ would be wrong.**
2. $2^5 \times 2^3 = 2^8 = 256$ **. Both forms are correct; the index form shows the structure and the number shows the size.**

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UNIT 1

ANSWER KEY — 3.1.5 Laws of Indices

Teachers: expansion turns each law into counting

C. PROVING THE LAWS BY EXPANDING

expand, and the laws stop being rules to memorise

$x^3 \times x^4 = (x \cdot x \cdot x)(x \cdot x \cdot x \cdot x) = x^7$

3 factors then 4 more — so 7 altogether: ADD

$y^5 \div y^2 = \frac{y \cdot y \cdot y \cdot y \cdot y}{y \cdot y} = y^3$

2 factors cancel from 5 — so 3 are left: SUBTRACT

$(c^2)^3 = c^2 \cdot c^2 \cdot c^2 = (c \cdot c)(c \cdot c)(c \cdot c) = c^6$

3 copies of 2 factors — so 6: MULTIPLY

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the base never changes — only the count of factors does

1. $a^3 \times a^2 = (a \cdot a \cdot a)(a \cdot a) = a^5$ — so we add the exponents.
2. Cancel 2 factors from 5, leaving b^3 .
3. $(c^2)^3 = c^2 \cdot c^2 \cdot c^2 = (cc)(cc)(cc) = c^6$.
4. $(p^2)^5$ means five *copies* of p^2 multiplied together, and each copy contributes 2 factors: $5 \times 2 = 10$ factors in all, so p^{10} . The wrong answer p^7 comes from adding 2 and 5, which is the rule for *multiplying* two separate powers — but here the 5 counts copies, not factors.

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UNIT 1

ANSWER KEY — 3.1.5 Laws of Indices

Teachers: x^7 / x^7 is 1, not 0 • brackets first, then subtract

D. WORKED FRAMES

1. $a^6 \times a^2 = a^8$
2. $b^9 \div b^4 = b^5$
3. $(c^3)^4 = c^{12}$
4. $2^5 \times 2^3 = 2^8 = 256$
5. $x^7 \div x^7 = x^0 = 1$ **— anything divided by itself is 1, which is the quotient law giving x^0 .**
6. $(2^2)^3 = 2^6 = 64$
7. **e.g.** $a^6 \times a^2 = (aaaaaa)(aa) = a^8$ **— six factors followed by two more makes eight.**

E. INDEPENDENT PRACTICE: CHOOSE THE CORRECT LAW

1. $m^4 \times m^7 = m^{11}$
2. $p^{12} \div p^5 = p^7$
3. $(q^4)^3 = q^{12}$
4. $5^0 = 1$
5. $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$
6. $(2^3)^4 \div 2^5 = 2^{12} \div 2^5 = 2^7$ **(= 128). Deal with the bracket first, then subtract.**

Name: _____ Date: _____

UNIT 1

ANSWER KEY — 3.1.5 Laws of Indices

Teachers: the power reaches BOTH letters • five broken laws

F. MIXED PRACTICE

1. $r^{10} \div r^6 \times r^2 = r^4 \times r^2 = r^6$ — **left to right.**
2. $(a^2)^3 \times a = a^6 \times a^1 = a^7$ — **remember the lone** a **is** a^1 **.**
3. $10^{-3} = \frac{1}{1000} = 0.001$
4. $(x^5y^3)^2 = x^{10}y^6$ — **the power reaches both letters (Section M).**
5. $4^{-2} = \frac{1}{16}$ **and** $4^0 = 1$ **, and** $\frac{1}{16} < 1$ **. A negative index gives a value *below* 1 for any base greater than 1, while a zero index sits exactly at 1 — the pattern is still dividing as the exponent falls.**
6. $(1+1)^2 = 4$ **but** $1^2 + 1^2 = 2$ **. Since** $4 \neq 2$ **, no — squaring does not distribute over addition.**

G. FIND AND CORRECT THE ERROR

1. **Ada — she used the power-of-a-power rule on a *product*. Multiplying powers means adding:** $x^3 \times x^5 = x^8$ **. (Expanded: 3 factors then 5 more makes 8.)**
2. **Ben — cancelling leaves 1, not 0. Any non-zero quantity divided by itself is 1:** $x^5 \div x^5 = x^0 = 1$ **.**
3. **Cara — she added instead of multiplying.** $(x^3)^4$ **is four copies of** x^3 **, so** $4 \times 3 = 12$ **factors:** x^{12} **.**
4. **Dev — the bases 2 and 3 are different, so no law applies. Work each power out:** $2^3 \times 3^2 = 8 \times 9 = 72$ **. (For comparison,** $6^5 = 7776$ **— nowhere near.)**
5. **Eze — a negative index does not make the answer negative; it means one over the power.** $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$ **.**

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UNIT 1

ANSWER KEY — 3.1.5 Laws of Indices

Teachers: a negative index divides, it does not negate

H. ZERO AND NEGATIVE INDICES

keep dividing by 3 and the pattern walks straight past zero

3^3	=	27	
3^2	=	9	÷ 3
3^1	=	3	÷ 3
3^0	=	1	÷ 3
3^{-1}	=	1/3	÷ 3
3^{-2}	=	1/9	÷ 3

$a^0 = 1 \cdot a^{-n} = 1 / a^n$ for every base except 0

a negative index does NOT mean a negative answer

it means one divided by the power — $3^{-2} = 1/9$, never -9

1. 27, 9, 3, 1 — each step divides by 3.
2. $3^{-1} = \frac{1}{3}$, $3^{-2} = \frac{1}{9}$ — the pattern simply keeps dividing.
3. $5^0 = 1 \cdot 2^{-3} = \frac{1}{8} \cdot 10^{-2} = \frac{1}{100} = 0.01$
4. $a^0 = 1$ and $a^{-n} = \frac{1}{a^n}$, provided $a \neq 0$.
5. **No.** $3^{-2} = \frac{1}{9}$, which is positive. A negative index reverses the *direction* of the pattern — it divides instead of multiplying — it does not change the sign. A negative **base** changes the sign; a negative **index** takes the reciprocal.
6. Because two rules collide. $0^n = 0$ for every $n > 0$ suggests 0, while $a^0 = 1$ for every non-zero base suggests 1. Neither has the stronger claim, so 0^0 is left undefined. Note this is exactly the case the quotient law excludes with its $a \neq 0$ condition.

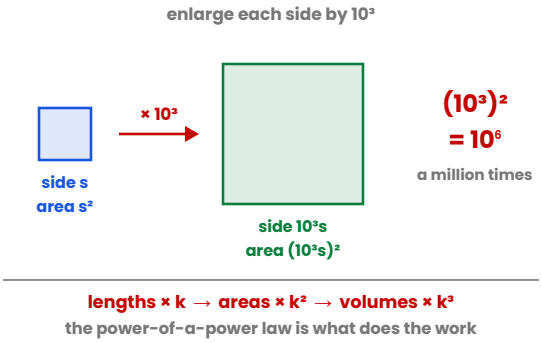
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UNIT 1

ANSWER KEY — 3.1.5 Laws of Indices

Teachers: 6 + 3 - 4 beats counting fifteen zeros

I. REAL-WORLD CONNECTION: SCALING AND POWERS OF TEN



1. $(10^3)^2 = 10^6$ — **the area becomes one million times the original.**
2. $(10^3)^3 = 10^9$ — **a thousand million times.**
3. $(10^6 \times 10^3) \div 10^4 = 10^9 \div 10^4 = 10^5$.
4. **Written out:** $(1\,000\,000 \times 1\,000) \div 10\,000$ **= 1 000 000 000 ÷ 10 000 = 100 000. The index version needs only** $6 + 3 - 4 = 5$ **— three small additions instead of counting fifteen zeros, with no chance of losing one.**
5. $10^8 \div 10^5 = 10^3$ • $(10^2)^4 = 10^8$.

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UNIT 1

ANSWER KEY — 3.1.5 Laws of Indices

Teachers: J4 is true once by accident - still not a law

J. INQUIRY: ALWAYS, SOMETIMES OR NEVER?

1. **Sometimes.** Required condition: the two powers must have the **same base.** $2^3 \times 2^4 = 2^7$ works; $2^3 \times 3^4$ cannot be combined at all.
2. **Always.** $(a^m)^n$ is n copies of a block of m factors, giving mn factors however the letters are chosen.
3. **Sometimes** — it needs $a \neq 0$. For every other base it always holds, including when $m < n$, where it produces a negative index rather than breaking down.
4. **Sometimes,** but only by accident. With $a = 2, m = n = 1$: $2^1 + 2^1 = 4$ and $2^{1+1} = 4$ — they agree! But with $m = 3, n = 2$: $8 + 4 = 12$ while $2^5 = 32$. One lucky case is not a law — which is exactly why a single counter-example settles it.
5. **Always.** $(ab)^n$ is n copies of ab, and multiplication can be reordered to collect the as and the bs separately: $a^n b^n$.

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UNIT 1

ANSWER KEY — 3.1.5 Laws of Indices

Teachers: the product law is the easiest to justify • a negative index turns a fraction upside down

K. EXTRA PRACTICE

1. $z^7 \times z^3 = z^{10}$
2. $b^9 \div b^4 = b^5$
3. $7^0 = 1$
4. $2^{-4} = \frac{1}{16}$
5. $(t^3)^2 \times t^4 = t^6 \times t^4 = t^{10}$
6. **Most students find the product law clearest: writing out a^3 and a^2 side by side and counting the factors makes the addition obvious in one line. The quotient law is a close second, since crossing off matching factors is just as visual. The power-of-a-power law needs one extra idea — copies of blocks — which is why it is the one most often got wrong.**

L. EXTENSION: NEGATIVE INDICES FROM THE QUOTIENT LAW

1. **By cancelling: $\frac{1}{a^2}$ (three factors cancel, two remain underneath). By the law: a^{-2}.**
   Both describe the same quantity, so $a^{-2} = \frac{1}{a^2}$. The negative index is not a new definition — it is what the quotient law is forced to mean.
2. $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$
3. $10^{-1} = 0.1$ • $10^{-2} = 0.01$ • $10^{-6} = 0.000001$
4. $5^{-1} = \frac{1}{5}$ • $3^{-3} = \frac{1}{27}$ • $\left(\frac{1}{2}\right)^{-1} = 2$ • $\left(\frac{1}{2}\right)^{-2} = 4$
5. **It turns the fraction upside down. $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n}$, because taking the reciprocal of a fraction swaps its numerator and denominator. That is why a negative index applied to a number below 1 makes it *bigger*.**
6. **$2^{-2} = \frac{1}{4}$, $2^{-1} = \frac{1}{2}$, $2^0 = 1$, $2^2 = 4$ → $2^{-2} < 2^{-1} < 2^0 < 2^2$. The exponents are in order and so are the values — the pattern runs smoothly through zero.**

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UNIT 1

ANSWER KEY — 3.1.5 Laws of Indices

Teachers: (2x)^3 is 8x^3 - the 2 is inside

M. EXTENSION: BRACKETS OVER PRODUCTS AND QUOTIENTS

a bracket shares the power with EVERY factor inside

$(ab)^n$	$= a^n b^n$	$(2x)^3 = 2^3x^3 = 8x^3$
$(a/b)^n$	$= a^n / b^n$	$(x/2)^3 = x^3/8$
$(x^5y^3)^2$	$= x^{10}y^6$	square BOTH exponents

$(2x)^3$ is $8x^3$, NOT $2x^3$ — the 2 is inside the bracket

but a bracket over a SUM shares nothing: $(a+b)^2$ is NOT $a^2 + b^2$

1. $(ab)^n = a^n b^n$ • $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
2. $(2x)^3 = 8x^3$ • $(3y)^2 = 9y^2$ • $(5m)^2 = 25m^2$
3. $(x^5y^3)^2 = x^{10}y^6$ • $(2a^2b)^3 = 8a^6b^3$
4. $\left(\frac{x}{2}\right)^3 = \frac{x^3}{8}$ • $\left(\frac{2}{3}\right)^2 = \frac{4}{9}$ • $\left(\frac{a^2}{b}\right)^3 = \frac{a^6}{b^3}$
5. **With $x = 1$: $(2 \times 1)^3 = 8$ but $2 \times 1^3 = 2$ — not equal. They forgot that the 2 is inside the bracket, so it gets cubed as well.**
6. **No. $(3+4)^2 = 49$ but $3^2 + 4^2 = 25$. A bracket shares a power over a product because multiplication can be reordered; it cannot do so over a sum, because there is nothing to reorder — expanding gives the extra middle term $2ab$.**

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UNIT 1

ANSWER KEY — 3.1.5 Laws of Indices

Teachers: numbers divide, exponents subtract • handle the digits and the powers separately

N. EXTENSION: SIMPLIFYING LONGER CHAINS

1. $x^5 \times x^3 \div x^2 = x^8 \div x^2 = x^6$
2. $(m^3)^2 \times m^4 \div m^5 = m^6 \times m^4 \div m^5 = m^5$ **— the exponent arithmetic is** $6 + 4 - 5$ **.**
3. $(2^3)^2 \div 2^4 = 2^6 \div 2^4 = 2^2 = 4$
4. $6a^5 \div 2a^2 = 3a^3$ • $12x^7 \div 4x^3 = 3x^4$ **— divide the numbers, subtract the exponents.**
5. $(3x^2)^2 \times 2x^3 = 9x^4 \times 2x^3 = 18x^7$
6. $(2p^3)^3 \div 4p^5 = 8p^9 \div 4p^5 = 2p^4$
7. **The numbers were divided normally (** $8 \div 4 = 2$**); the letters had their exponents subtracted (** $9 - 5 = 4$**). Index laws apply only to the letters — the coefficients follow ordinary arithmetic, and keeping the two jobs separate is what stops the whole thing going wrong.**

O. EXTENSION: INDEX LAWS WITH POWERS OF TEN

1. $10^6 \times 10^3 = 10^9$ • $10^9 \div 10^4 = 10^5$ • $(10^2)^4 = 10^8$
2. **Numbers:** $3 \times 2 = 6$ **. Powers:** $10^4 \times 10^3 = 10^7$ **. Answer:** 6×10^7 **(= 60 000 000).**
3. $(8 \div 2) \times (10^6 \div 10^2) = 4 \times 10^4$ **(= 40 000).**
4. $\frac{6 \times 10^{24}}{7.3 \times 10^{22}} = \frac{6}{7.3} \times 10^2 \approx 82$ **times heavier.**
   The index law did the hard part: it reduced $10^{24} \div 10^{22}$ **to** 10^2 **in one subtraction, leaving only the easy division** $6 \div 7.3$ **. Without it you would be cancelling twenty-two zeros by hand.**
5. $10^{-3} \times 10^5 = 10^{-3+5} = 10^2 = 100$ **— the law works exactly the same way with negative exponents.**

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UNIT 1

ANSWER KEY — 3.1.5 Laws of Indices

Teachers: exponents, same base, multiply or divide

P. EXTENSION: WHAT IS *NOT* AN INDEX LAW

four things that look like laws and are not

- ✗ $(m + n)^2 = m^2 + n^2$ — 1 + 1 gives 4, not 2
- ✗ $a^m + a^n = a^{m+n}$ — $2^3 + 2^2 = 12$, but $2^5 = 32$
- ✗ $2^3 \times 3^2 = 6^5$ — 72, not 7776 — different bases
- ✗ $3^{-2} = -9$ — $3^{-2} = 1/9$

one counter-example is enough to kill a rule for ever

the laws only ever combine EXPONENTS, and only for the SAME BASE

1. **LHS =** $(3+4)^2 = 49$ **, RHS =** $9+16 = 25$ **. False.**
2. **LHS =** $2^3 + 2^2 = 8+4 = 12$ **, RHS =** $2^5 = 32$ **. False.**
3. **LHS =** $8 \times 9 = 72$ **, RHS =** $6^5 = 7776$ **. False — and spectacularly so.**
4. $3^{-2} = \frac{1}{9}$ **, a small positive number.**
5. **Every one of them tries to apply a law where its condition fails. The laws combine exponents, for a single shared base, joined by multiplication or division. Rules 1 and 2 break it by using addition; rule 3 by using two different bases; rule 4 by treating the minus sign as belonging to the value rather than to the exponent. Whenever a step feels like a law, check those three words: exponents, same base, multiply or divide.**

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UNIT 1

ANSWER KEY — 3.1.5 Laws of Indices

Teachers: (4) fractional indices are forced, not invented

Q. CHALLENGE: PROVING AND USING THE LAWS

(1) a^m **is** m **factors of** a **;** a^n **is** n **factors. Multiplying them writes all of them in one product, giving** $m+n$ **factors, which is** a^{m+n} **.**
Note the proof never uses what a **is — only that the *same* a appears in both, which is precisely the same-base condition.**

(2) $a^n \div a^n$ **is 1 by cancelling and** a^0 **by the law, so** $a^0 = 1$ **.**
Then $a^0 \div a^n = \frac{1}{a^n}$ **by the first result, and** $a^{0-n} = a^{-n}$ **by the law, so** $a^{-n} = \frac{1}{a^n}$ **.**
Both "extra" rules are consequences of the quotient law, not separate facts to learn.

(3) $2^x = 32 = 2^5 \rightarrow x = 5$ • $3^x = 1 = 3^0 \rightarrow x = 0$ • $5^x = \frac{1}{25} = 5^{-2} \rightarrow x = -2$ **.**
$2^x \times 2^3 = 2^{x+3} = 2^7 \rightarrow x + 3 = 7 \rightarrow x = 4$ **.**
$(2^x)^3 = 2^{3x} = 2^{12} \rightarrow 3x = 12 \rightarrow x = 4$ **.**
The method throughout: rewrite both sides to the same base, then compare exponents.

(4) $(a^{1/2})^2 = a^1 = a$ **.**
So $a^{1/2}$ **is the number that squares to give** a **— that is,** $\sqrt{a}$ **.**
Therefore $9^{1/2} = 3$ **and** $8^{1/3} = 2$ **.**
This is the pattern of the whole unit: the laws are extended to new kinds of exponent by insisting they keep working, and each time the only consistent meaning turns out to be forced.