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UNIT 2

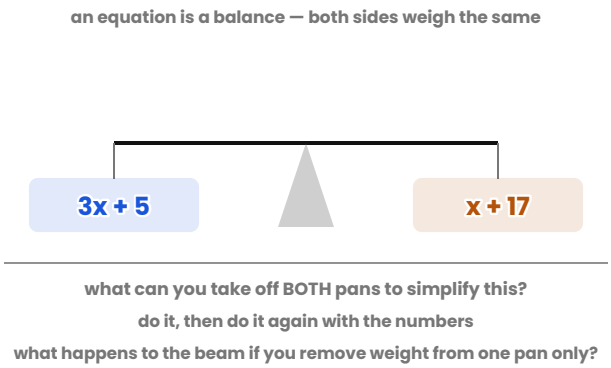
3.2.3 — Equations with Variables on Both Sides

IB Maths MYP • Year 3 • Unit 2

Variables to one side • Numbers to the other • Same operation on both sides

A. KEY VOCABULARY: KEEPING THE EQUATION BALANCED

An equation says two expressions have the same value. Whatever you do to one side you must do to the other, or the two sides stop matching.



1. **In** $3x + 5 = x + 17$ **, subtract** x **from both sides:** _____
2. **Now subtract 5 from both sides:** _____
3. **Finally divide by 2:** x **=** _____
4. **What happens to the balance if you remove weight from one pan only?**

   _____
5. **Solve** $3x + 4 = x + 12$ **:** _____

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UNIT 2

3.2.3 — Equations with Variables on Both Sides

Which Operation First?

B. WARM-UP: WHICH OPERATION FIRST?

Name the operation that removes the variable from one side, then solve.

Equation	First operation	Solution
$4x + 3 = x + 18$		
$7y - 5 = 3y + 15$		
$2a + 9 = 5a - 6$		
$4x + 6 = 2x + 18$		
$6x + 4 = 2x + 20$		

1. **In row 3 you could subtract $2a$ or $5a$. Which is easier, and why?** _____
2. **All five solutions are close together. Does that mean the method is easy, or that the questions were chosen kindly?** _____

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UNIT 2

3.2.3 — Equations with Variables on Both Sides

Solving Step by Step

C. SOLVING STEP BY STEP

Collect the variables on one side, the numbers on the other, then divide. Write the operation next to each line so the reasoning is visible.

one operation per line, both sides every time

5x − 7 = 2x + 11

? what did you do?

? what did you do?

? what did you do?

---

write the operation you used beside each line
then check your answer in the ORIGINAL equation
why subtract 2x rather than 5x?

1. $5x - 7 = 2x + 11$. **Subtract** $2x$ **:** _____
2. **Add 7:** _____ **Divide by 3:** x **=** _____
3. **Check by substituting: LHS** _____ **, RHS** _____
4. **Why subtract** $2x$ **rather than** $5x$ **?** _____

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UNIT 2

3.2.3 — Equations with Variables on Both Sides

Worked Frames

D. WORKED FRAMES

Show every inverse operation.

1. $4x + 3 = x + 18 \rightarrow$ _____ $\rightarrow x =$ _____
2. $7y - 5 = 3y + 15 \rightarrow$ _____ $\rightarrow y =$ _____
3. $2a + 9 = 5a - 6 \rightarrow$ _____ $\rightarrow a =$ _____
4. $3x + 4 = x + 14$: $x =$ _____
5. $6x + 4 = 2x + 20$: $x =$ _____
6. **Why must the same operation be done to both sides?**

_____

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UNIT 2

3.2.3 — Equations with Variables on Both Sides

Independent Practice • Solve and Check

E. INDEPENDENT PRACTICE

Check: show each inverse operation and check one answer by substitution.

1. $3x + 8 = x + 20$: x = _____
2. $9y - 4 = 5y + 16$: y = _____
3. $2p + 7 = 6p - 9$: p = _____
4. $8a - 3 = 3a + 22$: a = _____
5. $4m + 15 = 2m + 27$: m = _____
6. **Explain why the same operation must be performed on both sides:**

_____

F. MIXED PRACTICE: SOLVE AND CHECK

Show the balance-preserving steps, then check one answer in the original equation.

1. $5x + 2 = 3x + 14$: x = _____
2. $7t - 9 = 2t + 16$: t = _____
3. **Check (1) by substitution: LHS** _____ **, RHS** _____
4. $4r + 11 = 9r - 4$: r = _____
5. **Two plans cost** $12 + 3n$ **and** $4 + 5n$ **. When do they cost the same?** n = _____ **, cost** _____
6. **What does it mean if both sides simplify to the same expression?**

_____

Name: _____ Date: _____

UNIT 2

3.2.3 — Equations with Variables on Both Sides

Error Analysis

G. FIND AND CORRECT THE ERROR

Each student has made one mistake. Say what is wrong, then correct it.

1. **Ada: "** $4x + 7 = 2x + 15$ **, so** $2x = 8$ **— I took the 7 off the left."**
   Error: _____ **Correction:** _____
2. **Ben: "** $5x - 7 = 2x + 11$ **, so** $3x - 7 = 11$ **, so** $3x = 4$ **."**
   Error: _____ **Correction:** _____
3. **Cai: "** $2p + 7 = 6p - 9$ **, so** $8p = -2$ **."**
   Error: _____ **Correction:** _____
4. **Dee: "** $3x = 18$ **, so** $x = 18 - 3 = 15$ **."**
   Error: _____ **Correction:** _____
5. **Eli: "** $6x + 4 = 2x + 20$ **. I cancelled the** x **from the left, so** $6 + 4 = 2x + 20$ **."**
   Error: _____ **Correction:** _____

Name: _____ Date: _____

UNIT 2

3.2.3 — Equations with Variables on Both Sides

Checking by Substitution

H. CHECKING BY SUBSTITUTION

Work out each side separately and compare. Always use the **original** equation, not a later line of your working.

put the answer back into the ORIGINAL equation

5x + 2 = 3x + 14, x = 6

LEFT SIDE	RIGHT SIDE
5(6) + 2	3(6) + 14
= ?	= ?

work out each side separately

why check in the ORIGINAL equation and not in a later line?

1. $5x + 2 = 3x + 14$ **with** $x = 6$ **: LHS** _____ **, RHS** _____ **, so** _____

2. **Check** $x = 6$ **in** $3x + 5 = x + 17$ **: LHS** _____ **, RHS** _____

3. **Check** $a = 5$ **in** $2a + 9 = 5a - 6$ **: LHS** _____ **, RHS** _____

4. **Someone checks** $x = 5$ **in** $5x + 2 = 3x + 14$ **: LHS** _____ **, RHS** _____ **. What does that tell them?** _____

5. **Why check in the original rather than in a later line of working?**

_____

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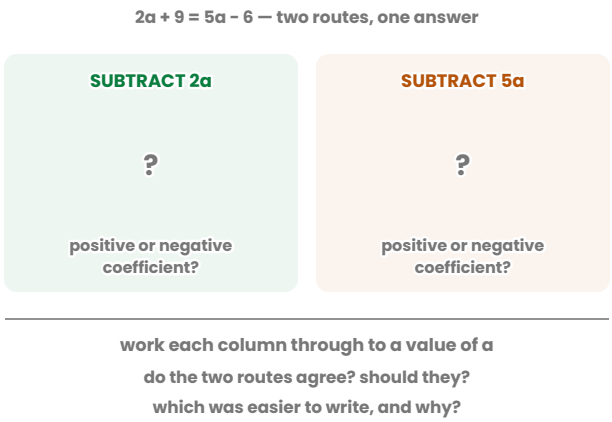
UNIT 2

3.2.3 — Equations with Variables on Both Sides

Choosing a Useful Order

I. CHOOSING A USEFUL ORDER

Either variable term can be removed first. Both routes give the same answer — one just has fewer minus signs to trip over.



1. **Subtract** $2a$ **from** $2a + 9 = 5a - 6$ **:** _____ → a **=** _____

2. **Now subtract** $5a$ **instead:** _____ → a **=** _____

3. **Did the two routes agree?** _____ **Should they?** _____

4. **Which was easier to write, and why?** _____

5. **State the rule of thumb: subtract the term with the** _____

Name: _____ Date: _____

UNIT 2

3.2.3 — Equations with Variables on Both Sides

Inquiry

J. INQUIRY: ALWAYS, SOMETIMES OR NEVER?

Circle your answer, then give a counterexample for anything not always true.

1. **"If the same value is added to both sides, the equation stays balanced."**
   Always / Sometimes / Never because _____
2. **"A variable can be cancelled from only one side."**
   Always / Sometimes / Never Counterexample: _____
3. **"A solution must make both sides equal."**
   Always / Sometimes / Never because _____
4. **"An equation with the variable on both sides has exactly one solution."**
   Always / Sometimes / Never Counterexample: _____
5. **"The solution of an equation must be a positive whole number."**
   Always / Sometimes / Never Counterexample: _____

Name: _____ Date: _____

UNIT 2

3.2.3 — Equations with Variables on Both Sides

Extra Practice

K. EXTRA PRACTICE

1. $3x + 5 = x + 17$: x = _____
2. $6y - 4 = 2y + 16$: y = _____
3. **Explain why both sides must receive the same operation:**

   _____
4. **Check** $x = 6$ **in** $3x + 5 = x + 17$: _____
5. **Two taxi firms charge** $20 + 4d$ **and** $8 + 6d$ **. Equal at** d = _____ **, fare**

   _____
6. **Where could two expressions for the same quantity be compared in real life?**

   _____

Name: _____ Date: _____

UNIT 2

3.2.3 — Equations with Variables on Both Sides

Negative and Fractional Solutions • Brackets on Both Sides

L. EXTENSION: NEGATIVE AND FRACTIONAL SOLUTIONS

Nothing in the method assumes the answer is a tidy positive whole number. The same steps work, and the check still settles it.

1. $2x + 9 = 5x + 21$: x = _____
2. $4x - 3 = 6x + 5$: x = _____
3. $7 - 2x = 3x + 22$: x = _____
4. $3x + 1 = x + 8$: x = _____
5. $5x + 2 = 2x + 4$: x = _____
6. **Check your answer to (1) by substitution: LHS** _____ **, RHS** _____ **Why is checking more important here?** _____

M. EXTENSION: BRACKETS ON BOTH SIDES

Expand first, using 3.2.1, and the equation becomes one you already know how to solve. The brackets are a disguise, not a new problem.

1. $3(x + 4) = 2x + 18$ **: expand** _____ **, so** x **=** _____
2. $5(x + 2) = 3(x + 6)$ **: expand** _____ **, so** x **=** _____
3. $2(3x - 1) = 4(x + 3)$: x = _____
4. $4(2y - 3) = 2(3y + 1)$: y = _____
5. $3(x - 2) - 2(x + 1) = 5$: x = _____
6. **In (5), why must the** -2 **multiply both terms in the second bracket?**

_____

Name: _____ Date: _____

UNIT 2

3.2.3 — Equations with Variables on Both Sides

Clearing Fractions

N. EXTENSION: CLEARING FRACTIONS

A denominator is removed by multiplying **every** term by it — not just the fraction. With two denominators, multiply by their lowest common multiple.

multiply EVERY term by the denominator

½x + 3 = x − 1

what would clear the fraction?

?

which terms must be multiplied — all of them, or just the fraction?
solve the equation you get, then check it in the original
what would you multiply by if there were two denominators?

1. $\frac{x}{2} + 3 = x - 1$: multiply everything by _____ to get _____ , so x = _____
2. $\frac{x}{3} = \frac{x-4}{2}$: multiply by _____ , so x = _____
3. $\frac{x+1}{2} = \frac{x-3}{4}$: x = _____
4. $\frac{2x}{5} + 1 = \frac{x}{2}$: x = _____
5. Check (1) in the original: LHS _____ , RHS _____
6. What goes wrong if you multiply only the fraction and not the whole line?

_____

Name: _____ Date: _____

UNIT 2

3.2.3 — Equations with Variables on Both Sides

One, None, or Every Value

O. EXTENSION: ONE SOLUTION, NONE, OR EVERY VALUE

Sometimes the variable disappears entirely. What is left behind — a true statement or a false one — tells you how many solutions there are.

collect the variables and see what is left

3x + 4 = x + 10 ?
how many solutions?

2x + 3 = 2x + 5 ?
how many solutions?

2(x + 3) = 2x + 6 ?
how many solutions?

---

solve each one as far as it will go
in two of them the x disappears — what does that mean?
how many solutions does each equation have?

1. $3x + 4 = x + 10 \rightarrow$ _____ , so _____ solution(s)
2. $2x + 3 = 2x + 5 \rightarrow$ _____ . Is that true? _____ So _____ solution(s)
3. $2(x + 3) = 2x + 6 \rightarrow$ _____ . Is that true? _____ So _____ solution(s)
4. $5x - 2 = 5x + 2$: _____ $4(x - 1) = 4x - 4$: _____
5. What do the two special cases have in common before the variable disappears?

   _____
6. An expression that is true for *every* value has a name. What is it? _____

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UNIT 2

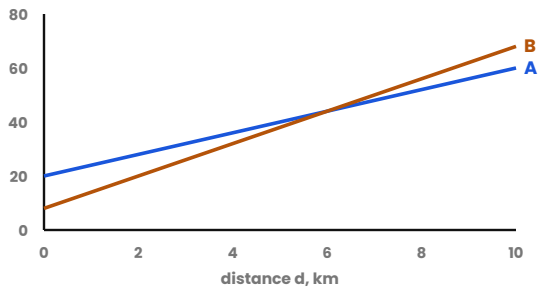
3.2.3 — Equations with Variables on Both Sides

Forming the Equation

P. EXTENSION: FORMING THE EQUATION YOURSELF

Two descriptions of the same quantity give an equation. Solving it finds the moment they agree — the point where two lines cross.

Company A: 20 + 4d • Company B: 8 + 6d



form an equation and solve it to find where the lines cross

which company is cheaper on a short trip? on a long one?

1. **Taxi A charges** $20 + 4d$ **, B charges** $8 + 6d$ **. Equation:** _____

2. **Solve it:** d **=** _____ **, fare** _____

3. **Which is cheaper for a 3 km trip?** _____ **For a 10 km trip?** _____

4. **Gym A costs** $30 + 5v$ **, gym B costs** $10 + 9v$ **, for** v **visits. Equal at** v **=** _____ **, cost** _____

5. **Two candles are** $30 - 3t$ **cm and** $12 - t$ **cm tall after** t **minutes. Same height at** t **=** _____ **min, height** _____ **cm**

6. **In (5), what is the meaning of the answer in the story?**

_____

Name: _____ Date: _____

UNIT 2

3.2.3 — Equations with Variables on Both Sides

Challenge

Q. CHALLENGE: REASONING ABOUT EQUATIONS

(1) Solve the general equation.

Starting from $ax + b = cx + d$, collect the x terms: _____

So x = _____

Test the formula on $5x - 7 = 2x + 11$: _____

(2) What the formula forbids.

The formula fails when _____. What are a and c then? _____

Connect this to section O: if $a = c$ and $b = d$ there are _____ solutions; if $a = c$ but $b \neq d$ there are _____

(3) When is the solution negative?

From your formula, x is negative when _____

Build an equation of the form $ax + b = cx + d$ whose solution is -4: _____ Check it: _____

(4) A crossover with no meaning.

Gym A costs $10 + 8v$ and gym B costs $20 + 9v$. Solve: v = _____

Why can that answer not describe a real membership? _____

So which gym is cheaper, and for how many visits? _____

ANSWER KEY

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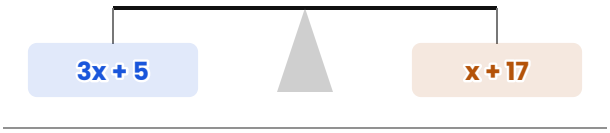
UNIT 2

ANSWER KEY — 3.2.3 Equations with Variables on Both Sides

Teachers: remove from one pan and the beam tips

A. KEY VOCABULARY: KEEPING THE EQUATION BALANCED

an equation is a balance — both sides weigh the same



take x off BOTH pans: 2x + 5 = 17
take 5 off BOTH pans: 2x = 12, so x = 6

remove the same weight from one pan only and the beam tips
that is the whole reason for doing the same thing to both sides

1. $2x + 5 = 17$.
2. $2x = 12$.
3. $x = 6$.
4. **The beam tips — the two sides are no longer equal, so the new equation has a different solution from the old one. Every legitimate step preserves the truth of the statement, and that is the only reason the answer at the end is the answer to the question you started with.**
5. $3x + 4 = x + 12 \rightarrow 2x + 4 = 12 \rightarrow 2x = 8 \rightarrow x = 4$.

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UNIT 2

ANSWER KEY — 3.2.3 Equations with Variables on Both Sides

Teachers: the deck's answers are all 4, 5 or 6

B. WARM-UP: WHICH OPERATION FIRST?

Equation	First operation	Solution
$4x + 3 = x + 18$	- x	$x = 5$
$7y - 5 = 3y + 15$	- 3y	$y = 5$
$2a + 9 = 5a - 6$	- 2a	$a = 5$
$4x + 6 = 2x + 18$	- 2x	$x = 6$
$6x + 4 = 2x + 20$	- 2x	$x = 4$

1. Subtracting $2a$ is easier: it leaves $9 = 3a - 6$ with a positive coefficient. Subtracting $5a$ leaves $-3a + 9 = -6$, which is correct but invites a sign slip. Both give $a = 5$.
2. The questions were chosen kindly — all five answers are 4, 5 or 6. Real equations produce negatives and fractions just as often; sections L and N supply them.

Name: _____ Date: _____

UNIT 2

ANSWER KEY — 3.2.3 Equations with Variables on Both Sides

Teachers: subtract the smaller x-term

C. SOLVING STEP BY STEP

one operation per line, both sides every time

5x − 7 = 2x + 11

3x − 7 = 11 — − 2x from both sides

3x = 18 — + 7 to both sides

x = 6 — ÷ 3 on both sides

---

check: 5(6) − 7 = 23 and 2(6) + 11 = 23

subtracting the SMALLER x-term first keeps the coefficient positive

variables to one side, numbers to the other, then divide

1. $3x - 7 = 11$.
2. $3x = 18$, so $x = 6$.
3. **LHS =** $5(6) - 7 = 23$, **RHS =** $2(6) + 11 = 23$ **— equal, so the solution is right.**
4. **Because taking away the *smaller* variable term leaves a positive coefficient on the other side. Subtracting** $5x$ **instead gives** $-7 = -3x + 11$ **, which is perfectly correct but carries two extra minus signs through the rest of the working — and minus signs are where the marks go.**

Name: _____ Date: _____

UNIT 2

ANSWER KEY — 3.2.3 Equations with Variables on Both Sides

Teachers: every step keeps the statement true • the solution set never changes

D. WORKED FRAMES

1. $3x + 3 = 18 \rightarrow 3x = 15 \rightarrow x = 5$.
2. $4y - 5 = 15 \rightarrow 4y = 20 \rightarrow y = 5$.
3. $9 = 3a - 6 \rightarrow 15 = 3a \rightarrow a = 5$.
4. $2x + 4 = 14 \rightarrow x = 5$.
5. $4x + 4 = 20 \rightarrow 4x = 16 \rightarrow x = 4$.
6. **Because an equation is a statement that two quantities are equal. Changing one side alone makes that statement false, so every later line would be about a different problem. Doing the same thing to both keeps the statement true — which is why the value you reach at the end still satisfies the equation you began with.**

E. INDEPENDENT PRACTICE

1. $2x + 8 = 20 \rightarrow x = 6$.
2. $4y - 4 = 16 \rightarrow y = 5$.
3. $7 = 4p - 9 \rightarrow 16 = 4p \rightarrow p = 4$.
4. $5a - 3 = 22 \rightarrow 5a = 25 \rightarrow a = 5$.
5. $2m + 15 = 27 \rightarrow m = 6$.
6. **Because the equals sign is a claim that the two sides have the same value. An operation applied to only one side breaks that claim: it produces a new equation with a different solution. Doing the same to both keeps the balance level, so the solution set never changes as you work.**

Name: _____ Date: _____

UNIT 2

ANSWER KEY — 3.2.3 Equations with Variables on Both Sides

Teachers: F6 points at identities - see O • Dee subtracted where she should divide

F. MIXED PRACTICE: SOLVE AND CHECK

1. $2x + 2 = 14 \rightarrow x = 6$.
2. $5t - 9 = 16 \rightarrow 5t = 25 \rightarrow t = 5$.
3. **LHS =** $5(6) + 2 = 32$ **, RHS =** $3(6) + 14 = 32$ **— correct.**
4. $11 = 5r - 4 \rightarrow 15 = 5r \rightarrow r = 3$.
5. $12 + 3n = 4 + 5n \rightarrow 8 = 2n \rightarrow n = 4$ **, and both plans then cost** $12 + 12 = 24$.
6. **That the equation is an identity — true for every value of the variable, not just one. Section O works through examples of this.**

G. FIND AND CORRECT THE ERROR

1. **Ada: she removed the 7 from the left only, which tips the balance. Subtract** $2x$ **from both sides to get** $2x + 7 = 15$ **, then subtract 7 from both:** $2x = 8$ **, so** $x = 4$ **. (Her answer happens to be right, but the reasoning that produced it is not.)**
2. **Ben: to undo "** -7 **" you must add 7, not subtract it.** $3x - 7 = 11$ **gives** $3x = 18$ **and** $x = 6$.
3. **Cai: he added the two variable terms instead of subtracting one from the other. Subtracting** $2p$ **gives** $7 = 4p - 9$ **, then** $16 = 4p$ **and** $p = 4$.
4. **Dee:** $3x$ **means** $3 \times x$ **, so the inverse is division, not subtraction.** $x = 18 \div 3 = 6$.
5. **Eli: you cannot cancel a variable from one side alone — and** $6x$ **is not "** 6 **and an** x **" to be separated. Subtract** $2x$ **from both sides:** $4x + 4 = 20$ **, so** $x = 4$.

Name: _____ Date: _____

UNIT 2

ANSWER KEY — 3.2.3 Equations with Variables on Both Sides

Teachers: check the ORIGINAL, not your working

H. CHECKING BY SUBSTITUTION

put the answer back into the ORIGINAL equation

5x + 2 = 3x + 14, x = 6

LEFT SIDE	RIGHT SIDE
5(6) + 2	3(6) + 14
= 32	= 32

32 = 32, so x = 6 is correct

check in the ORIGINAL — a slip in line one would survive later

1. **LHS 32 , RHS 32 , so $x = 6$ is correct.**
2. **LHS = 23 , RHS = 23 — correct.**
3. **LHS = 19 , RHS = 19 — correct.**
4. **LHS = 27 , RHS = 29 . They differ, so $x = 5$ is not a solution — there is a mistake somewhere in the working, and the check has caught it.**
5. **Because a mistake made in the first line would be carried into every line after it. Checking against a later line would only confirm that you copied your own error faithfully; only the original equation is independent of your working.**

Name: _____ Date: _____

UNIT 2

ANSWER KEY — 3.2.3 Equations with Variables on Both Sides

Teachers: both routes are valid, one is tidier

I. CHOOSING A USEFUL ORDER

2a + 9 = 5a - 6 — two routes, one answer

SUBTRACT 2a	SUBTRACT 5a
9 = 3a - 6	-3a + 9 = -6
15 = 3a	-3a = -15
a = 5	a = 5
coefficient stays positive	coefficient stays negative

both routes are correct and both give a = 5

but subtracting the SMALLER coefficient avoids the minus signs

a preference, not a rule — the maths does not care

1. $9 = 3a - 6 \rightarrow 15 = 3a \rightarrow a = 5$.
2. $-3a + 9 = -6 \rightarrow -3a = -15 \rightarrow a = 5$.
3. **Yes , and they must — both routes are valid balance-preserving steps applied to the same equation, so they cannot lead to different answers.**
4. **The first, because it never produced a negative coefficient. The second required dividing -15 by -3 , which is where a sign is most often lost.**
5. **... subtract the term with the smaller coefficient, so that the variable ends up on the side where its coefficient stays positive. It is a preference for readability, not a mathematical requirement.**

Name: _____ Date: _____

UNIT 2

ANSWER KEY — 3.2.3 Equations with Variables on Both Sides

Teachers: J4 - not every equation has one solution • two tariffs, one crossing point

J. INQUIRY: ALWAYS, SOMETIMES OR NEVER?

1. **Always.** If two quantities are equal, adding the same amount to each keeps them equal. The same holds for subtracting, and for multiplying or dividing by any non-zero number.
2. **Never.** Counterexample: from $6x + 4 = 2x + 20$, deleting the x on the left alone gives $6 + 4 = 2x + 20$, whose solution is $x = -5$ — not the correct $x = 4$. "Cancelling" is really subtracting, and subtraction happens on both sides.
3. **Always.** That is precisely the definition of a solution: the value that makes the two sides agree. It is also why substitution is a complete check.
4. **Sometimes.** Counterexample: $2x + 3 = 2x + 5$ has no solution, and $2(x + 3) = 2x + 6$ has infinitely many.
5. **Sometimes.** Counterexample: $2x + 9 = 5x + 21$ gives $x = -4$, and $3x + 1 = x + 8$ gives $x = 3.5$. The deck's examples are unusually tidy.

K. EXTRA PRACTICE

1. $2x + 5 = 17 \rightarrow x = 6$.
2. $4y - 4 = 16 \rightarrow y = 5$.
3. Because the equals sign asserts that both sides have the same value. An operation on one side alone destroys that, and every line afterwards would be solving a different equation.
4. $3(6) + 5 = 23$ and $6 + 17 = 23$ — equal.
5. $20 + 4d = 8 + 6d \rightarrow 12 = 2d \rightarrow d = 6$ km, and both fares are then $20 + 24 = 44$.
6. Comparing two tariffs, two subscription plans, two job offers with different base pay and commission, hiring against buying, or two routes with different fixed and per-kilometre costs. In each case one expression describes each option and the equation finds the point at which the choice flips.

Name: _____ Date: _____

UNIT 2

ANSWER KEY — 3.2.3 Equations with Variables on Both Sides

Teachers: a wrong negative answer does not look wrong • the minus multiplies the whole bracket

L. EXTENSION: NEGATIVE AND FRACTIONAL SOLUTIONS

1. $9 = 3x + 21 \rightarrow -12 = 3x \rightarrow x = -4$.
2. $-3 = 2x + 5 \rightarrow -8 = 2x \rightarrow x = -4$.
3. $7 = 5x + 22 \rightarrow -15 = 5x \rightarrow x = -3$.
4. $2x + 1 = 8 \rightarrow 2x = 7 \rightarrow x = \frac{7}{2} = 3.5$.
5. $3x + 2 = 4 \rightarrow 3x = 2 \rightarrow x = \frac{2}{3}$.
6. **LHS =** $2(-4) + 9 = 1$ **, RHS =** $5(-4) + 21 = 1$ **— correct. Checking matters more here because a single lost minus sign changes the answer completely and, unlike a tidy positive whole number, a wrong negative answer does not *look* wrong.**

M. EXTENSION: BRACKETS ON BOTH SIDES

1. $3x + 12 = 2x + 18$ **, so** $x = 6$.
2. $5x + 10 = 3x + 18 \rightarrow 2x = 8 \rightarrow x = 4$.
3. $6x - 2 = 4x + 12 \rightarrow 2x = 14 \rightarrow x = 7$.
4. $8y - 12 = 6y + 2 \rightarrow 2y = 14 \rightarrow y = 7$.
5. $3x - 6 - 2x - 2 = 5 \rightarrow x - 8 = 5 \rightarrow x = 13$.
6. **Because** $-2(x + 1)$ **means** -2 **times the whole bracket. Multiplying only the** x **would give** $-2x + 1$ **and lose 2 from the left-hand side, changing the answer. This is the commonest error in the section, and it is the distributive law from 3.2.1 with a negative in front.**

Name: _____ Date: _____

UNIT 2

ANSWER KEY — 3.2.3 Equations with Variables on Both Sides

Teachers: multiply EVERY term, not just the fraction

N. EXTENSION: CLEARING FRACTIONS

multiply EVERY term by the denominator

½x + 3 = x - 1

↓ × 2 everywhere

x + 6 = 2x - 2

8 = x

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check: ½(8) + 3 = 7 and 8 - 1 = 7

the 3 and the 1 get multiplied too — EVERY term, not just the fraction

with two denominators, multiply by their lowest common multiple

1. **Multiply by 2 to get** $x+6=2x-2$ **, so** $x=8$ **.**
2. **Multiply by 6 :** $2x=3(x-4) \rightarrow 2x=3x-12 \rightarrow x=12$ **.**
3. **Multiply by 4:** $2(x+1)=x-3 \rightarrow 2x+2=x-3 \rightarrow x=-5$ **.**
4. **Multiply by 10:** $4x+10=5x \rightarrow x=10$ **.**
5. **LHS =** $\frac{8}{2}+3=7$ **, RHS =** $8-1=7$ **— correct.**
6. **You would be multiplying one side by 2 and the other by 1, which unbalances the equation. In (1), doubling only the fraction gives** $x+3=x-1$ **, which has no solution at all — a clear signal that something went wrong. Every term, on both sides, gets multiplied.**

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Teachers: what is left after x vanishes decides

O. EXTENSION: ONE SOLUTION, NONE, OR EVERY VALUE

collect the variables and see what is left

3x + 4 = x + 10	→	2x = 6	x = 3
		ONE solution	
2x + 3 = 2x + 5	→	3 = 5	false
		NO solution	
2(x + 3) = 2x + 6	→	0 = 0	always true
		EVERY value	

the x terms vanish in rows 2 and 3 — what is LEFT decides

a false statement means no value works; a true one means every value does

row 3 is an IDENTITY, not really an equation to solve

1. $2x = 6$, so $x = 3$ — one solution.
2. $3 = 5$, which is **false**, so there are **no** solutions. No value of x can make a number three larger than itself equal to one five larger.
3. $6 = 6$, or $0 = 0$, which is **true**, so **every** value works. Expanding the left gives exactly the right-hand side.
4. $5x - 2 = 5x + 2$ gives $-2 = 2$ — no solution. $4(x - 1) = 4x - 4$ gives $0 = 0$ — every value.
5. In both, the two sides have the **same coefficient of x** , so the variable cancels completely. What remains is a statement about numbers only: if it is true the equation was an identity, and if it is false the two sides can never meet.
6. An **identity** — and it is usually written with $\equiv$ rather than = to signal that it holds for all values. Every factorisation in 3.2.2 was an identity in this sense.

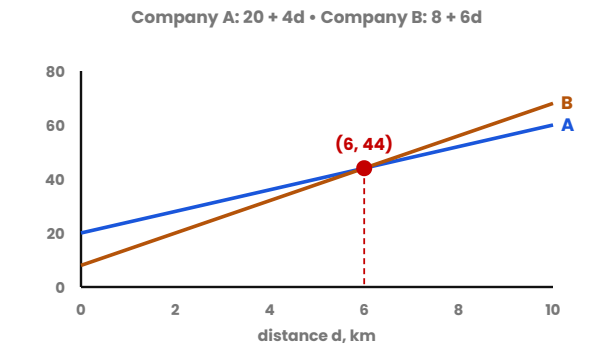
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UNIT 2

ANSWER KEY — 3.2.3 Equations with Variables on Both Sides

Teachers: the crossing point is where the choice flips

P. EXTENSION: FORMING THE EQUATION YOURSELF



20 + 4d = 8 + 6d → 12 = 2d → d = 6, fare 44

solving the equation FINDS the crossing point; B is cheaper before it, A after

1. $20 + 4d = 8 + 6d$.
2. $12 = 2d$, so $d = 6$ km and the fare is 44 for both.
3. At 3 km: A costs 32, B costs 26, so B is cheaper. At 10 km: A costs 60, B costs 68, so A is cheaper. The crossing point is exactly where the better choice changes.
4. $30 + 5v = 10 + 9v \rightarrow 20 = 4v \rightarrow v = 5$ visits, costing 55 .
5. $30 - 3t = 12 - t \rightarrow 18 = 2t \rightarrow t = 9$ minutes, at height 3 cm .
6. The taller candle burns faster, so after 9 minutes the two are level at 3 cm. After that the first is *shorter* than the second. Note the answer is only meaningful while both candles still exist: the first reaches zero at $t = 10$, so the model expires shortly after the crossing point.

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ANSWER KEY — 3.2.3 Equations with Variables on Both Sides

Teachers: (4) a valid solution that answers nothing

Q. CHALLENGE: REASONING ABOUT EQUATIONS

(1) Subtracting cx and b from both sides gives $ax - cx = d - b$, that is $(a-c)x = d - b$.
So $x = \frac{d-b}{a-c}$.
For $5x - 7 = 2x + 11$: $\frac{11-(-7)}{5-2} = \frac{18}{3} = 6$ — matching the answer found step by step.

(2) The formula fails when the denominator is zero, that is when $a = c$ — the two sides have the same coefficient of x.
If $a = c$ and $b = d$ the two sides are identical, so there are **infinitely many** solutions. If $a = c$ but $b \neq d$ there are **none.** Section O's three cases are exactly the three things the formula can do: give a value, divide by zero with equal numerators, or divide by zero with unequal ones.

(3) x is negative when $d - b$ and $a - c$ have **opposite signs** — when the side with the bigger x coefficient also has the bigger constant.
For example $2x + 9 = 5x + 21$: $\frac{21-9}{2-5} = \frac{12}{-3} = -4$.
Check: $2(-4) + 9 = 1$ and $5(-4) + 21 = 1$ — correct.

(4) $10 + 8v = 20 + 9v \rightarrow -10 = v$, so $v = -10$.
A membership cannot involve a negative number of visits, so the crossing point lies outside the range the model describes. The algebra is fine; the *context* restricts v to whole numbers from 0 upwards.
Since gym A is cheaper at $v = 0$ (10 against 20) and the lines never cross again for positive v, **A is cheaper for every possible number of visits.** This is worth noticing: an equation can have a perfectly good solution that answers no question anyone asked.