Name: _____ Date: _____

UNIT 2

2.2.5 — Two-step Equations

Inverse Operations • Balance • Negative Coefficients • Checking by Substitution • Algebraic Reasoning

A. CORE METHOD

Work backwards through the operations surrounding the variable. Keep both sides of the equation balanced.

General structure

$ax + b = c \Rightarrow ax = c - b \Rightarrow x = \frac{c-b}{a}$

Example

$3x + 5 = 20 \Rightarrow 3x = 15 \Rightarrow x = 5$

Check: $3(5) + 5 = 20$

1. For **3x + 5 = 20**, which operation should be undone first? _____
2. Explain why inverse operations are applied in reverse order.

   _____
3. Complete: Whatever operation is performed on one side of an equation must also be performed on _____ .
4. Why is substitution a useful final check?

   _____

B. SOLVE TWO-STEP EQUATIONS

Show the inverse operation used at each step.

1. **2x + 7 = 19** _____
2. **5y - 3 = 22** _____
3. **-3a + 4 = 19** _____
4. **4x + 3 = 23** _____
5. **6y - 8 = 22** _____
6. **-4n - 6 = 14** _____

Name: _____ Date: _____

UNIT 2

2.2.5 — Two-step Equations

Accuracy with Signs • Substitution • Error Analysis • Justifying the Order of Operations

C. NEGATIVE COEFFICIENTS AND CHECKS

When dividing by a negative number, track the sign carefully. Check the starred equations by substitution.

1. **7x - 5 = 30** _____
2. ★ **3a + 14 = 2** _____
3. **-5y + 10 = 35** _____
4. **8p + 4 = 52** _____
5. **2m - 13 = 1** _____
6. ★ **-3r - 9 = 12** _____

D. FIND, CORRECT AND EXPLAIN

1. A student writes: **4x + 7 = 23** → **4x = 30** → **x = 7.5**. Identify the first error and correct the solution.

   _____

2. A student solves **-2x + 7 = 15** and reaches **-2x = 8**, then writes **x = 4**. Explain the sign error and give the correct value.

   _____

3. **Always, sometimes or never?** "In a two-step equation, you always add or subtract before multiplying or dividing." Test the claim using an equation of the form **ax + b = c** and a counterexample such as $\frac{x}{4} + 2 = -16$ or $\frac{x+6}{5} = 8$.

   _____

   _____

4. Explain why changing only one side of an equation can change its solution set.

   _____

Name: _____ **Date:** _____

UNIT 2

2.2.5 — Two-step Equations

Model Contexts • Form Equations • Fractions and Decimals • Interpret Solutions

E. EQUATIONS IN CONTEXT

Define the variable, write the equation, solve it and interpret the answer in context.

1. Three notebooks cost **x** dollars each and delivery costs $5. The total is $26. Write and solve the equation.

   _____

2. A museum charges a $12 booking fee plus $11 per student. A group pays $78. How many students are in the group?

   _____

3. Five identical game tokens and a $7 service fee cost $32 in total. Find the cost of one token.

   _____

4. “Three less than five times a number is 42.” Write an equation and solve it.

   _____

5. A temperature starts at 11°C and decreases by 4°C each hour. After **h** hours it is -9°C. Find **h**.

   _____

F. FRACTIONS AND DECIMALS

These remain two-step equations, but the inverse operation may involve multiplying by a denominator or dividing by a decimal coefficient.

1. $\frac{x}{4} + 2 = -16$ _____
2. **(2/3)x - 5 = 9** _____
3. **-1.5m + 4.2 = 13.2** _____
4. **0.4p - 1.8 = 4.6** _____
5. Choose one equation above and verify the result by substitution.

Name: _____ Date: _____

UNIT 2

2.2.5 — Two-step Equations

Extended Problems • Reverse Reasoning • Structure • Generalisation

G. EXTENDED PROBLEMS — MULTI-STEP REASONING

These problems extend the lesson into less familiar representations while remaining linear.

1. **Membership model.** A gym charges a one-time joining fee of $18 plus the same monthly fee each month. After 7 months, the total paid is $151. Find the monthly fee.

2. **Delivery model.** Eight identical science kits plus a $10 delivery charge cost $146. Find the price of one kit.

3. **Negative multiplier.** A number is multiplied by -4, then 9 is added. The result is -35. Find the number.

4. **Grouped operations.** The result of adding 6 to a number and then dividing by 5 is 8. Write an equation and solve it.

H. ANALYSIS AND TRANSFER

5. **Compare methods.** For **5x - 7 = 18**, one student adds 7 to both sides and then divides by 5. Another divides each side by 5 first and then adds 7. Explain which reasoning preserves equivalence and why.

6. **Construct.** Write a two-step equation with a negative coefficient whose solution is **x = -6**. Then show that your equation is correct by substitution.

7. **Unknown coefficient.** The equation **kx + 5 = 20** has solution **x = 3**. Find **k** and justify your answer.

8. **Always, sometimes or never?** "If the coefficient of the variable is negative, then the solution must be negative." Justify with examples or counterexamples.

ANSWER KEY

Name: _____ Date: _____

UNIT 2

ANSWER KEY — 2.2.5 Two-step Equations

Equivalent algebraic working earns full credit when balance and inverse operations are shown clearly.

A. CORE METHOD

1. **Subtract 5 from both sides.**
2. **Solving reverses the sequence used to build the expression; undo the outermost/last operation first.**
3. **the other side**.
4. **Substitution checks whether the proposed value makes the original equation true.**

B. SOLVE TWO-STEP EQUATIONS

1. **x = 6**
2. **y = 5**
3. **a = -5**
4. **x = 5**
5. **y = 5**
6. **n = -5**

C. NEGATIVE COEFFICIENTS AND CHECKS

1. **x = 5**
2. **a = -4**. Check: $3(-4) + 14 = 2$.
3. **y = -5**
4. **p = 6**
5. **m = 7**
6. **r = -7**. Check: $-3(-7) - 9 = 12$.

Name: _____ Date: _____

UNIT 2

ANSWER KEY — 2.2.5 Two-step Equations

Teachers: accept equivalent verbal explanations where the mathematical reasoning is explicit.

D. FIND, CORRECT AND EXPLAIN

1. The first error is arithmetic: **23 - 7 = 16**, not 30. Then 4x = 16, so **x = 4**.
2. Dividing 8 by -2 gives a negative value. Correct: **x = -4**.
3. **Sometimes.** For ax + b = c, undo b before dividing by a. But in (x + 6)/5 = 8, multiply by 5 first, then subtract 6. The correct order depends on how the operations are nested.
4. Changing only one side usually destroys equality; **equivalent equations are produced by applying the same reversible operation to both sides**.

E. EQUATIONS IN CONTEXT

1. 3x + 5 = 26, so 3x = 21 and **x = 7 dollars**.
2. 11s + 12 = 78, so 11s = 66 and **s = 6 students**.
3. 5t + 7 = 32, so 5t = 25 and **t = 5 dollars**.
4. 5x - 3 = 42, so 5x = 45 and **x = 9**.
5. 11 - 4h = -9, so -4h = -20 and **h = 5 hours**.

F. FRACTIONS AND DECIMALS

1. x/4 + 2 = -16 → x/4 = -18 → **x = -72**.
2. (2/3)x - 5 = 9 → (2/3)x = 14 → **x = 21**.
3. -1.5m + 4.2 = 13.2 → -1.5m = 9 → **m = -6**.
4. 0.4p - 1.8 = 4.6 → 0.4p = 6.4 → **p = 16**.
5. Substitution should reproduce the original right-hand side; for example, 0.4(16) - 1.8 = **4.6**.

Name: _____ Date: _____

UNIT 2

ANSWER KEY — 2.2.5 Two-step Equations

Extended reasoning may be shown using equations, balance models, inverse-operation arrows, or substitution.

G. EXTENDED PROBLEMS — WORKED ANSWERS

1. $7m + 18 = 151 \rightarrow 7m = 133 \rightarrow$ **m = 19 dollars per month**.
2. $8k + 10 = 146 \rightarrow 8k = 136 \rightarrow$ **k = 17 dollars**.
3. $-4x + 9 = -35 \rightarrow -4x = -44 \rightarrow$ **x = 11**.
4. $(x + 6)/5 = 8 \rightarrow x + 6 = 40 \rightarrow$ **x = 34**.

H. ANALYSIS AND TRANSFER — WORKED ANSWERS

5. The first method preserves equivalence directly: add 7 to both sides, then divide both sides by 5. Dividing "each side by 5 first" is only valid if the entire left side is divided: **(5x - 7)/5**, not just 5x/5. The correct solution is **x = 5**.
6. Answers vary. Example: **-3x + 4 = 22**. Then $-3x = 18$, $x = -6$; check: $-3(-6) + 4 = 22$.
7. Substitute $x = 3$: $3k + 5 = 20 \rightarrow 3k = 15 \rightarrow$ **k = 5**.
8. **Sometimes.** Example with a negative solution: $-2x + 7 = 15$ gives $x = -4$. Example with a positive solution: $-2x + 7 = -1$ gives $x = 4$.

EXTENSION NOTE

The challenge set extends the source lesson into fractional and decimal coefficients, grouped operations, contextual modelling, unknown coefficients and counterexample reasoning. These are intended to deepen transfer without moving into multi-variable or quadratic equations.