Name: _____ Date: _____

UNIT 2

1.2.6 — Number Sequences

Terms • Position • Vocabulary

A. KEY VOCABULARY

Complete each definition using precise mathematical language.

1. A **sequence** is _____.
2. A **term** is _____.
3. The **position** tells us _____.
4. The **common difference** is _____.
5. The **common multiplier (ratio)** is _____.
6. An **nth-term rule** gives _____.

Arithmetic: $a_n = a_1 + (n-1)d$

Geometric: $a_n = a_1 r^{n-1}$

A. QUICK CHECK

Write one arithmetic sequence with common difference -4 and one geometric sequence with common multiplier 1/2.

_____

Name: _____ Date: _____

UNIT 2

1.2.6 — Number Sequences

Arithmetic & Geometric Patterns • Classification

B. EXTEND AND CLASSIFY

For each sequence, decide whether it is arithmetic or geometric, state the common difference/multiplier, and find the next two terms.

Sequence	Type	Difference / multiplier	Next two terms
4, 7, 10, 13, …			
12, 9, 6, 3, …			
2, 6, 18, 54, …			
80, 40, 20, 10, …			
-3, 1, 5, 9, …			
3, -6, 12, -24, …			

B. QUICK CHECK

Classify each: 2, 5, 8, 11, … and 3, 6, 12, 24, … . State the difference or multiplier.

_____

Name: _____ Date: _____

UNIT 2

1.2.6 — Number Sequences

Position • Terms • Rules • Representations

C. POSITION, TERMS, AND RULES

Complete the table. Then describe how each rule changes from one position to the next.

Position, *n*	**1**	**2**	**3**	**4**	**5**	**8**
$a_n = 3n + 1$						
$b_n = 2 \cdot 3^{n-1}$						

Explain why the first rule is arithmetic but the second is geometric.

_____

_____

C. QUICK CHECK

Using the rules in the table, find a_6 and b_4 without extending every term.

_____

Name: _____ Date: _____

UNIT 2

1.2.6 — Number Sequences

Translate Patterns into nth-Term Rules

D. TRANSLATE A PATTERN INTO A RULE

Write an nth-term rule. For arithmetic sequences, show the form first term + (n - 1)(difference).

1. 7, 11, 15, 19, ... Rule: _____
2. 22, 17, 12, 7, ... Rule: _____
3. 3, 9, 27, 81, ... Rule: _____
4. 1/2, 1, 2, 4, ... Rule: _____

D. QUICK CHECK

A sequence starts at 5 and increases by 6 each term. Write an nth-term rule.

_____

Name: _____ Date: _____

UNIT 2

1.2.6 — Number Sequences

Substitution • Reverse Problems • nth-Term Rules

E. USE A RULE - AND WORK BACKWARDS

1. $a_n = 5n - 1$. Find a_9. _____
2. $p_n = 2n + 1$. Find p_{13}. _____
3. $q_n = 4n - 3$. Write the first three terms. _____
4. $a_n = 5 + 4(n - 1)$. If $a_n = 81$, find n. _____
5. $b_n = 3 \cdot 2^{n-1}$. Find b_6. _____
6. $c_n = 81\left(\frac{1}{3}\right)^{n-1}$. Find c_5. _____

E. QUICK CHECK

If $a_n = 4n - 3$ and $a_n = 45$, find n.

_____

Name: _____ Date: _____

UNIT 2

1.2.6 — Number Sequences

Error Analysis • Checking Rules

F. FIND AND CORRECT THE ERROR

A student sees the sequence 3, 7, 11, 15, … and writes the rule below.

> Student rule: $a_n = 3n + 7$

1. Check the rule at $n = 1$. What value does it give? _____
2. Explain why this proves the rule is incorrect.

   _____
3. Write the correct rule in two equivalent forms.

   _____
4. Another student says 2, 6, 18, 54 is arithmetic because the terms are increasing. Explain the error.

   _____

F. QUICK CHECK

For the sequence 5, 9, 13, 17, …, verify that $a_n = 4n + 1$ works for $n = 1$ and $n = 4$.

_____

Name: _____ **Date:** _____

UNIT 2

1.2.6 — Number Sequences

Repeated Change • Real-World Modelling

G. SEQUENCES MODEL REPEATED CHANGE

1. A stadium has 20 seats in row 1 and 5 more seats in each new row.
   (a) Write the first five terms. _____
   (b) Write a rule for row n. _____
   (c) Find the number of seats in row 8. _____
   (d) Which row has 70 seats? _____
2. A savings plan has $12 after week 1 and increases by $7 each week.
   (a) Write a rule for the amount in week n. _____
   (b) Find the amount in week 12. _____
   (c) Explain why this model is arithmetic, not geometric.
   _____

G. QUICK CHECK

A theatre has 14 seats in row 1 and 3 more seats in each new row. How many seats are in row 10?

_____

Name: _____ **Date:** _____

UNIT 2

1.2.6 — Number Sequences

Reconstructing Sequences from Later Terms

H. EXTENDED: RECONSTRUCT THE SEQUENCE

These problems give information about later terms instead of the first term.

1. An arithmetic sequence has $a_4 = 17$ and $a_{10} = 41$.
   (a) Find the common difference. _____
   (b) Find the first term. _____
   (c) Write an nth-term rule. _____
   (d) Find a_{25}. _____
2. A geometric sequence has $b_2 = 6$ and $b_5 = 162$ and has a positive multiplier.
   (a) Find the multiplier. _____
   (b) Find the first term. _____
   (c) Write an nth-term rule. _____
   (d) Find b_7. _____

H. QUICK CHECK

An arithmetic sequence has $a_4 = 17$ and $a_9 = 37$. Find the common difference.

_____

Name: _____ **Date:** _____

UNIT 2

1.2.6 — Number Sequences

Growing Designs • Generalization

I. GROWING DESIGN

A tiled design uses 8 tiles at stage 1 and gains 4 tiles at every new stage.

1. Complete: 8, 12, 16, _____ , _____ .
2. Write a rule for the number of tiles at stage *n*.

   _____
3. How many tiles are needed for stage 100? _____
4. At which stage are exactly 200 tiles used? Show how you know.

   _____

I. QUICK CHECK

A growing pattern follows $T_n = 5 + 3(n - 1)$. Find the number of tiles at stage 20.

_____

Name: _____ Date: _____

UNIT 2

1.2.6 — Number Sequences

Comparing Additive and Multiplicative Models

J. EXTENDED: COMPARE TWO MODELS

Model A begins at 10 and increases by 6 each stage. Model B begins at 2 and doubles each stage.

$A_n = 10 + 6(n - 1)$ $B_n = 2 \cdot 2^{n-1} = 2^n$

Stage *n*	**1**	**2**	**3**	**4**	**5**	**6**
Model A						
Model B						

At what first stage does Model B become larger than Model A? _____ Explain why doubling eventually grows faster than adding 6.

_____

J. QUICK CHECK

For the two models above, compare their values at stage 4 and state which is larger.

_____

Name: _____ Date: _____

UNIT 2

1.2.6 — Number Sequences

Reasoning • Examples • Counterexamples

K. ALWAYS, SOMETIMES, OR NEVER?

Classify each claim. Give an example or counterexample to justify your choice.

1. A sequence with a clear pattern has a constant difference. _____

   _____

2. A geometric sequence can also be arithmetic. _____

   _____

3. An arithmetic sequence with a positive common difference increases as n increases. _____

   _____

K. QUICK CHECK

Give one counterexample to the claim “every patterned sequence has a constant difference.”

_____

Name: _____ Date: _____

UNIT 2

1.2.6 — Number Sequences

Non-routine Challenge • Generalization

L. NON-ROUTINE CHALLENGE

1. An arithmetic sequence has 8th term 29 and 15th term 50.
   (a) Find the common difference. _____ (b) Find the first term. _____
   (c) Write the rule. _____
2. Is 100 a term of the sequence 7, 12, 17, 22, …? Justify your answer using an nth-term rule.

   _____
3. The geometric sequence 81, −27, 9, −3, … continues with the same multiplier.
   (a) State the multiplier. _____ (b) Find term 7. _____

L. QUICK CHECK

The sequence 5, 11, 17, 23, … has common difference 6. Determine whether 101 is a term.

_____

ANSWER KEY

Name: _____ Date: _____

UNIT 2

ANSWER KEY — 1.2.6 Number Sequences

Worked answers • Equivalent correct reasoning earns full credit.

A. KEY VOCABULARY

1. **An ordered list of terms.**
2. **One value in a sequence.**
3. **Where a term appears in the sequence.**
4. **The fixed amount added or subtracted between consecutive terms of an arithmetic sequence.**
5. **The fixed number each term is multiplied by to get the next term of a geometric sequence.**
6. **A rule that gives the value of the term at any position n.**

A. QUICK CHECK

Answer: Example: arithmetic 12, 8, 4, 0, … ; geometric 16, 8, 4, 2, …

Name: _____ Date: _____

UNIT 2

ANSWER KEY — 1.2.6 Number Sequences

Worked answers • Equivalent correct reasoning earns full credit.

B. EXTEND AND CLASSIFY

Sequence	Type	Difference / multiplier	Next two
4, 7, 10, 13	Arithmetic	+3	16, 19
12, 9, 6, 3	Arithmetic	−3	0, −3
2, 6, 18, 54	Geometric	×3	162, 486
80, 40, 20, 10	Geometric	×1/2	5, 2.5
−3, 1, 5, 9	Arithmetic	+4	13, 17
3, −6, 12, −24	Geometric	×(−2)	48, −96

B. QUICK CHECK

Answer: 2, 5, 8, 11, … is arithmetic with d = 3. 3, 6, 12, 24, … is geometric with multiplier 2.

Name: _____ Date: _____

UNIT 2

ANSWER KEY — 1.2.6 Number Sequences

Worked answers • Equivalent correct reasoning earns full credit.

C. POSITION, TERMS, AND RULES

$a_n = 3n + 1$: **4, 7, 10, 13, 16, 25**

$b_n = 2 \cdot 3^{n-1}$: **2, 6, 18, 54, 162, 4374**

The first adds 3 for each increase of 1 in n. The second multiplies by 3.

C. QUICK CHECK

Answer: $a_6 = 3(6) + 1 = 19$. $b_4 = 2 \cdot 3^3 = 54$.

Name: _____ Date: _____

UNIT 2

ANSWER KEY — 1.2.6 Number Sequences

Worked answers • Equivalent correct reasoning earns full credit.

D. TRANSLATE A PATTERN INTO A RULE

1. $a_n = 7 + 4(n - 1) = 4n + 3$
2. $b_n = 22 - 5(n - 1) = 27 - 5n$
3. $c_n = 3 \cdot 3^{n-1} = 3^n$
4. $d_n = \frac{1}{2} \cdot 2^{n-1}$

D. QUICK CHECK

Answer: $a_n = 5 + 6(n - 1)$, equivalently $a_n = 6n - 1$.

Name: _____ Date: _____

UNIT 2

ANSWER KEY — 1.2.6 Number Sequences

Worked answers • Equivalent correct reasoning earns full credit.

E. USE A RULE - AND WORK BACKWARDS

1. **44**
2. **27**
3. **1, 5, 9**
4. **$5 + 4(n - 1) = 81 \Rightarrow 4(n - 1)=76 \Rightarrow n=20.$**
5. **96**
6. **1**

E. QUICK CHECK

Answer: **$4n - 3 = 45 \Rightarrow 4n = 48 \Rightarrow n = 12.$**

Name: _____ Date: _____

UNIT 2

ANSWER KEY — 1.2.6 Number Sequences

Worked answers • Equivalent correct reasoning earns full credit.

F. FIND AND CORRECT THE ERROR

1. **10.**
2. **At n=1 the sequence should give 3, so the proposed rule does not reproduce the first term.**
3. $a_n = 3 + 4(n - 1) = 4n - 1$
4. **Increasing terms do not prove a sequence is arithmetic. Here each term is multiplied by 3, so it is geometric.**

F. QUICK CHECK

Answer: $a_1 = 4(1)+1 = 5$ and $a_4 = 4(4)+1 = 17$, so the rule matches both checked terms.

Name: _____ Date: _____

UNIT 2

ANSWER KEY — 1.2.6 Number Sequences

Worked answers • Equivalent correct reasoning earns full credit.

G. SEQUENCES MODEL REPEATED CHANGE

1. **(a) 20, 25, 30, 35, 40. (b)** $S_n = 20 + 5(n - 1)$**. (c) 55 seats. (d) Row 11.**
2. **(a)** $M_n = 12 + 7(n - 1)$**. (b) $89. (c) The same amount, $7, is added each week; there is no constant multiplier.**

G. QUICK CHECK

Answer: $a_{10} = 14 + 3(10 - 1) = 41$ **seats.**

Name: _____ Date: _____

UNIT 2

ANSWER KEY — 1.2.6 Number Sequences

Worked answers • Equivalent correct reasoning earns full credit.

H. EXTENDED: RECONSTRUCT THE SEQUENCE

1. **From term 4 to term 10 there are 6 equal steps: d=(41-17)/6=4. Then a_1=17-3(4)=5. Rule:** $a_n = 4n + 1$**. a_{25}=101.**
2. **$b_5/b_2=r^3=162/6=27$, so r=3. Then b_1=2. Rule:** $b_n = 2 \cdot 3^{n-1}$**. b_7=1458.**

H. QUICK CHECK

Answer: Five steps separate terms 4 and 9, so d = (37 - 17)/5 = 4.

Name: _____ Date: _____

UNIT 2

ANSWER KEY — 1.2.6 Number Sequences

Worked answers • Equivalent correct reasoning earns full credit.

I. GROWING DESIGN

1. **20, 24.**
2. $T_n = 8 + 4(n - 1) = 4n + 4$
3. **T_{100}=4(100)+4=404 tiles.**
4. **4n+4=200 ⇒ 4n=196 ⇒ n=49.**

I. QUICK CHECK

Answer: **T_{20} = 5 + 3(19) = 62 tiles.**

Name: _____ Date: _____

UNIT 2

ANSWER KEY — 1.2.6 Number Sequences

Worked answers • Equivalent correct reasoning earns full credit.

J. EXTENDED: COMPARE TWO MODELS

Stage n	1	2	3	4	5	6
Model A	10	16	22	28	34	40
Model B	2	4	8	16	32	64

Model B first exceeds Model A at stage 6. Repeated multiplication makes the amount added between terms grow, whereas Model A always adds only 6.

J. QUICK CHECK

Answer: Model A: $10 + 6(3) = 28$. Model B: $2 \cdot 2^3 = 16$. Model A is larger at stage 4.

Name: _____ Date: _____

UNIT 2

ANSWER KEY — 1.2.6 Number Sequences

Worked answers • Equivalent correct reasoning earns full credit.

K. ALWAYS, SOMETIMES, OR NEVER?

1. **Sometimes.** Arithmetic examples have a constant difference, but 1, 2, 4, 8, … or 1, 4, 9, 16, … do not.
2. **Sometimes.** A constant sequence such as 5, 5, 5, … is arithmetic (d=0) and geometric (r=1).
3. **Always.** Adding a positive amount each step makes every next term larger.

K. QUICK CHECK

Answer: Example: 1, 2, 4, 8, … has a clear pattern but differences 1, 2, 4, … are not constant.

Name: _____ Date: _____

UNIT 2

ANSWER KEY — 1.2.6 Number Sequences

Worked answers • Equivalent correct reasoning earns full credit.

L. NON-ROUTINE CHALLENGE

1. **There are 7 steps from term 8 to term 15, so d=(50-29)/7=3. a_1=29-7(3)=8. Rule:** $a_n = 3n + 5$**.**
2. **Rule:** $a_n = 5n + 2$**. Solving 5n+2=100 gives n=19.6, not a whole-number position, so 100 is not a term.**
3. **Multiplier = -1/3. Term 7 = 1/9.**

L. QUICK CHECK

Answer: $a_n = 5 + 6(n - 1) = 6n - 1$. $6n - 1 = 101$ gives $n = 17$, so yes.