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UNIT 5

1.5.6 — Mean, Median, and Mode

Central Tendency • Ordered Data • Outliers • Choosing an Average • Comparing Data Sets

A. CENTRAL TENDENCY FOUNDATIONS

Complete each statement. Use precise mathematical language.

Mean formula: $\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n}$

1. The **mean** is found by adding _____ and dividing by _____ .
2. Before finding the **median**, the data must be _____ .
3. For an even number of values, the median is the _____ .
4. The **mode** is the value that _____ .
5. An extreme value usually affects the _____ more than the median because the mean uses _____ .

B. CALCULATE MEAN, MEDIAN AND MODE

Order the data for the median and show the total used for the mean.

Data set	Mean	Median	Mode
A: 4, 5, 5, 6, 10	_____	_____	_____
B: 2, 3, 3, 3, 4, 15	_____	_____	_____
C: 6, 7, 7, 8, 8, 9	_____	_____	_____
D: 3, 5, 7, 9, 11	_____	_____	_____

1. Which data set is **bimodal**? _____
2. Which data set has **no mode**? _____
3. In which data set is the mean pulled noticeably above the median by one large value? _____

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C. AVERAGES FROM A FREQUENCY TABLE

A score can appear more than once. Frequency tells you how many times it occurs.

Score, x	2	3	4	5	6
Frequency, f	1	3	4	2	2

Extension notation: $\bar{x} = \frac{\sum fx}{\sum f}$

1. How many observations are represented? _____
2. Find the total of all the scores. _____
3. Calculate the mean. Give your answer to 2 decimal places.

   _____
4. Find the median and mode. Median = _____ Mode = _____

D. OUTLIERS AND CHOOSING AN AVERAGE

A good statistical summary explains not only the calculation, but why the chosen measure is useful.

1. Test scores are **8, 9, 9, 10, 12, 20**.
   (a) Find the mean, median and mode.

   _____

   (b) Replace 20 by 11. Recalculate the mean and median.

   _____

   (c) Which measure changed more? Explain why.

   _____
2. A library records books read by five students: **2, 3, 3, 4, 18**.
   Find the mean and median, then decide which one better describes a typical student. Justify your choice.

   _____

   _____

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E. REVERSE AVERAGES AND MISSING VALUES

Work backwards from the total implied by the mean.

Useful structure: $x_{\text{missing}} = n\bar{x} - \sum x_{\text{known}}$

1. Five scores have mean 12. Four scores are 9, 11, 13 and 15. Find the missing score.

   _____

2. Six values have mean 8.5. A seventh value is added and the new mean is 9. Find the added value.

   _____

3. The data are **4, 6, 8, x, 10** and the mean is 7.2. Find x, then state the median and mode.

   _____

4. Seven quiz scores have mean 8 and mode 9. Five known scores are **6, 8, 9, 9, 10**. The lowest score was 5. Find the two missing scores.

F. COMPARING AND COMBINING GROUPS

Combined mean: $\bar{x}_{\text{combined}} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$

1. Class A has 12 students with mean score 14. Class B has 18 students with mean score 11. Find the mean score of all 30 students together.

   _____

2. Two delivery teams have times (minutes):
   Team P: **5, 6, 6, 7, 8** Team Q: **3, 5, 6, 7, 11**.
   (a) Show that both teams have the same mean.

   _____

   (b) Compare their medians and modes.

   _____

   (c) Explain why one average alone does not tell the whole story.

   _____

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G. EXTENDED PROBLEMS — MULTI-STEP REASONING

These questions require reverse reasoning, correction of data and combining information.

1. **Missing outlier.** The ordered data are **5, 8, 8, 9, 10, 12, x**. Their mean is 10.
   (a) Find x. (b) Find the median and mode. (c) Explain why the mean is larger than the median.

2. **Correcting an error.** Eight measurements have a reported mean of 14. Later, a value recorded as 38 is found to be 18. Find the corrected mean.

3. **Moving between groups.** Club A has 15 students with mean 6 books read; Club B has 10 students with mean 9 books read. A student who read 18 books moves from B to A.
   (a) Find the original combined mean. (b) Find the new mean for each club. (c) Does the combined mean change? Explain.

4. **Two conditions.** Six ordered values are **4, 6, x, 8, 10, y**. Their median is 7 and their mean is 8. Find x and y, then state the mode.

H. ANALYSIS, CONSTRUCTION AND COMMUNICATION

5. **Always, sometimes or never?** "If the mean equals the median, there cannot be an unusual extreme value." Classify the statement and support your answer with examples or counterexamples.

6. **Construct a data set.** Create five integers from 3 to 15 with mean 8, median 7 and mode 6.

7. **Responsible reporting.** Waiting times are **4, 5, 5, 6, 6, 7, 28** minutes. Calculate the mean and median. Which should a school report as a "typical" wait, and what would be misleading about quoting only the other measure?

ANSWER KEY

Name: _____ Date: _____

UNIT 5

ANSWER KEY — 1.5.6 Mean, Median, and Mode

Teachers: equivalent reasoning and correctly justified choices of average earn full credit.

A. CENTRAL TENDENCY FOUNDATIONS

1. Add **all data values** and divide by **the number of values**.
2. The data must be **ordered from least to greatest (or greatest to least)**.
3. The median is the **mean of the two middle ordered values**.
4. The mode **occurs most often**; there may be more than one mode or no mode.
5. The **mean** is usually affected most because it uses **every value and its magnitude**.

B. CALCULATE MEAN, MEDIAN AND MODE

Set	Mean	Median	Mode
A	**6**	**5**	**5**
B	**5**	**3**	**3**
C	**7.5**	**7.5**	**7 and 8**
D	**7**	**7**	**no mode**

1. Bimodal: **C**.
2. No mode: **D**.
3. Large value pulls mean above median: **B**.

C. AVERAGES FROM A FREQUENCY TABLE

1. Total frequency = **12**.
2. Total score = $2(1)+3(3)+4(4)+5(2)+6(2)$ = **49**.
3. Mean = $49/12$ = **4.08** to 2 d.p.
4. Positions 6 and 7 are both 4, so median = **4**; the most frequent score is 4, so mode = **4**.

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UNIT 5

ANSWER KEY — 1.5.6 Mean, Median, and Mode

Teachers: equivalent reasoning and correctly justified choices of average earn full credit.

D. OUTLIERS AND CHOOSING AN AVERAGE

1. Original: total 68, so mean = **11.33...**; median = **9.5**; mode = **9**. With 11 instead of 20: total 59, mean = **9.83...**; median remains **9.5**. The mean changes by 1.5 while the median does not, because the mean uses the size of every value.
2. Total = 30, so mean = **6**; median = **3**. The **median** better describes a typical student because 18 is an unusually high value that pulls the mean upward.

E. REVERSE AVERAGES AND MISSING VALUES

1. Required total = 5(12) = 60; known total = 48; missing score = **12**.
2. Old total = 6(8.5) = 51; new total = 7(9) = 63; added value = **12**.
3. Required total = 5(7.2) = 36; known total = 28, so x = **8**. Ordered data 4, 6, 8, 8, 10: median = **8**, mode = **8**.
4. Total must be 56. Known total is 42, so the two missing scores total 14. Since the lowest score is 5, one missing score is 5; the other is **9**. Missing scores: **5 and 9**, which makes 9 the unique mode.

F. COMPARING AND COMBINING GROUPS

1. Combined total = 12(14)+18(11)=366. Combined mean = 366/30 = **12.2**.
2. Team P total 32 and Team Q total 32, so both means are **6.4**. Both medians are **6**. P has mode **6**; Q has **no mode**. The sets have different shapes/spread even though their means and medians match, so one average cannot describe every feature.

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UNIT 5

ANSWER KEY — 1.5.6 Mean, Median, and Mode

Teachers: equivalent reasoning and correctly justified choices of average earn full credit.

G. EXTENDED PROBLEMS — WORKED ANSWERS

1. Seven values with mean 10 have total 70. Known total = 52, so x = **18**. Median = **9**; mode = **8**. The high value 18 pulls the mean upward.
2. Reported total = 8(14) = 112. Corrected total = 112-38+18 = 92. Corrected mean = **11.5**.
3. Original totals: A = 15(6)=90, B = 10(9)=90, total = 180 for 25 students, so combined mean = **7.2**. After the move: A total 108 for 16 students, mean = **6.75**; B total 72 for 9 students, mean = **8**. The combined mean stays **7.2** because no score was added or removed from the overall group.
4. Median = (x+8)/2 = 7, so x = **6**. Mean 8 gives total 48; known total including x is 34, so y = **14**. The mode is **6**.

H. ANALYSIS, CONSTRUCTION AND COMMUNICATION

5. **Sometimes.** Mean = median can occur in balanced data with no unusual extreme value, but it can also occur with extreme values that balance one another. Example: 0, 0, 5, 5, 15 has mean 5 and median 5 while 15 is unusually large compared with most values.
6. One valid set is **6, 6, 7, 10, 11**. The sum is 40, so mean = 8; median = 7; mode = 6.
7. Total = 61, so mean = **61/7 ≈ 8.71 min**; median = **6 min**. Report the median as the typical wait because 28 is an extreme long wait. Quoting only the mean would make most students' usual experience seem longer than it is, while quoting only the median should still be accompanied by mention of the rare long wait.

EXTENSION NOTE

The challenge set extends the slide content into frequency tables, missing-value constraints, corrected data, combined means, constructed data sets and responsible interpretation. These are original problems using common IB-style reasoning patterns rather than copied questions.