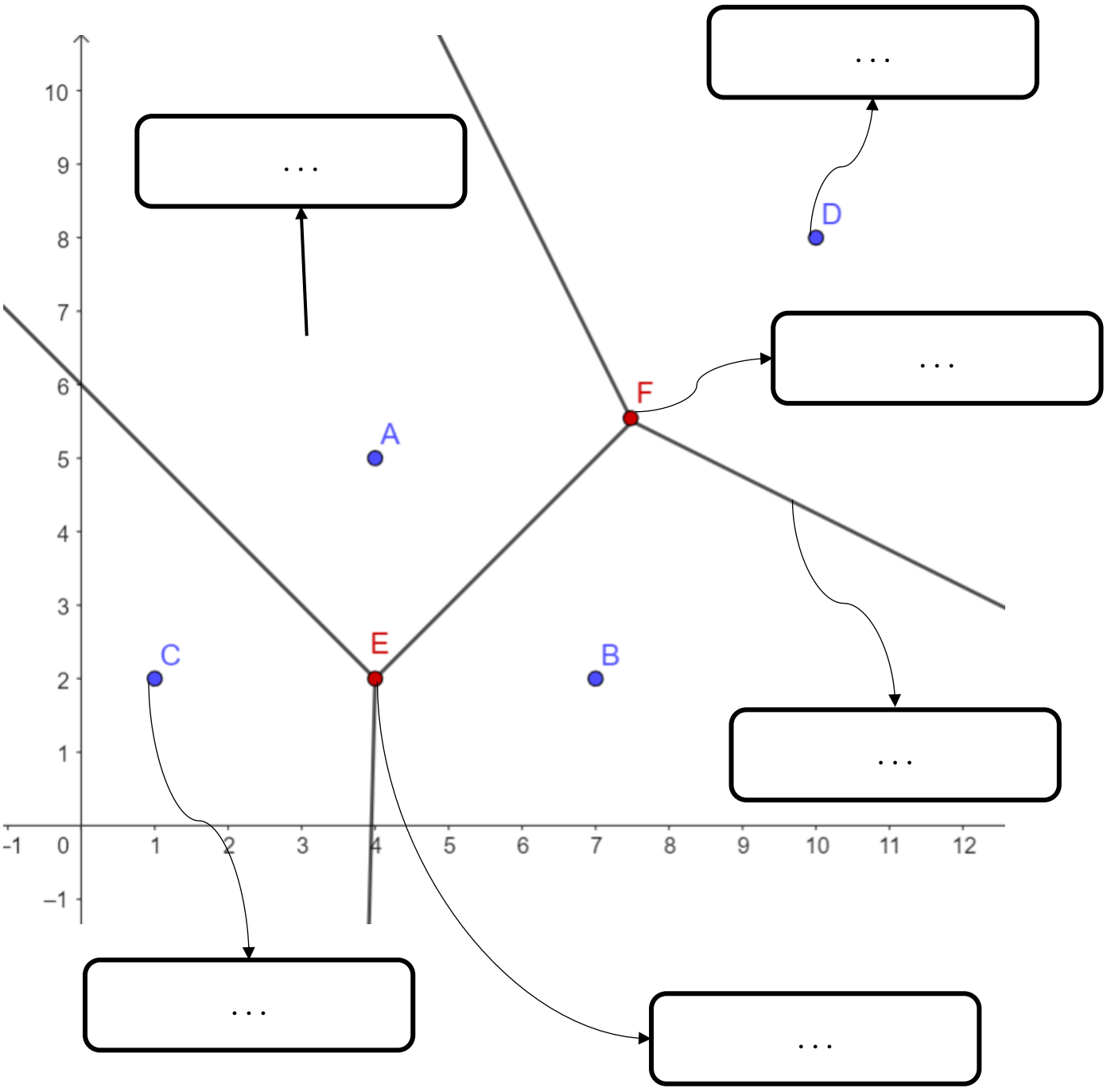
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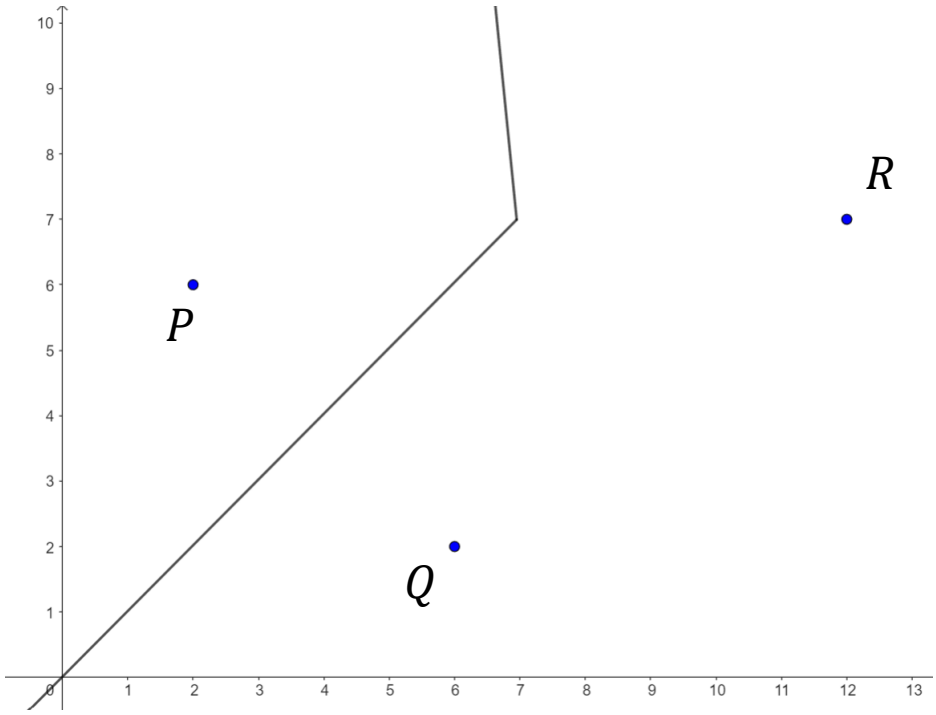
UNIT 3

Voronoi Diagrams

1. Write down the parts of the Voronoi diagram below.



2. A map of water towers in Sun City is shown in the figure below. Three towers, P, Q, and R, are located at the coordinates given.

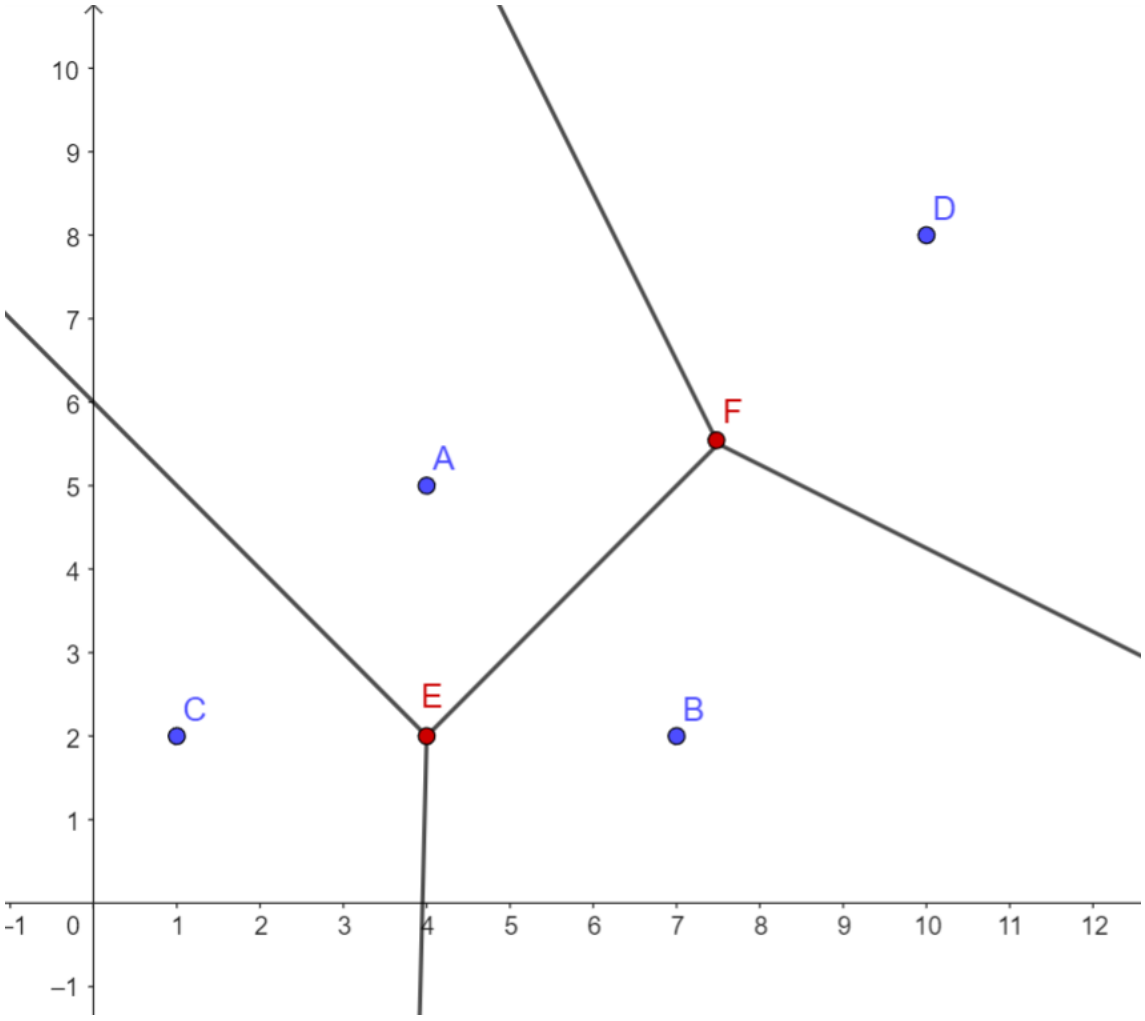


a) Find the gradient of segment QR.

b) Find the equation needed to complete the Voronoi diagram (in the form of $y = mx + c$).

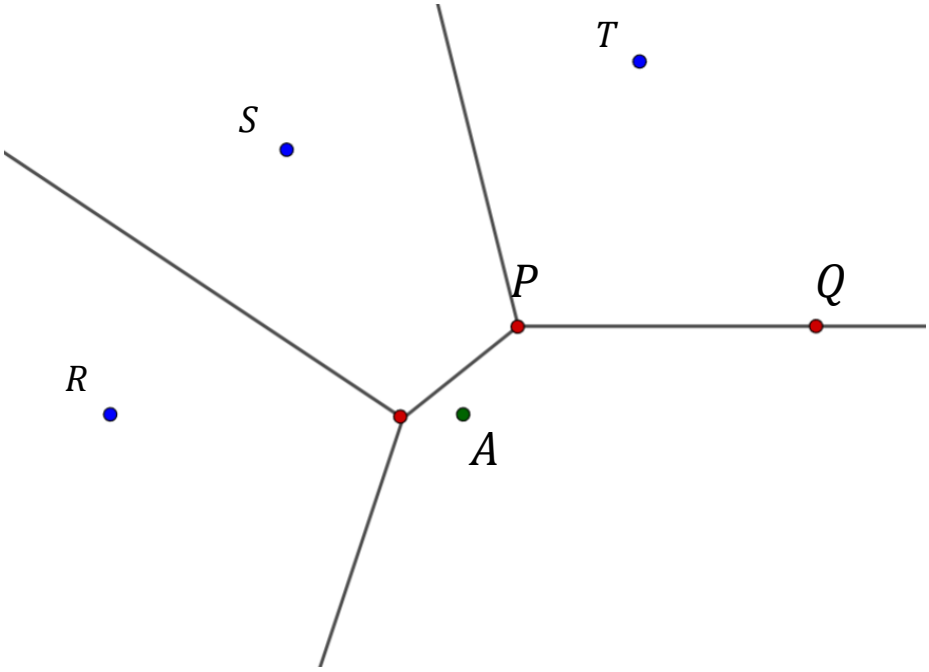
c) Draw the line to complete the diagram.

3. In the game *Clash of Kings*, players require not only attacking skills but also effective strategies for building defences. Thomas is working on constructing an archer tower as part of his defence strategy. The defence plan he developed is illustrated in the diagram below, where points A, B, C, and D represent various barbarian camps. Determine the optimal location for the tower to achieve the maximum shooting range, using the concept from the toxic waste dump problem. Explain your reasoning, and include a circle in the provided plan to represent the archer's shooting range.



4. Two pizza shops are located at coordinates $A(2,4)$ and $B(7,2)$. The plan is to build a third shop at coordinates $C(10,7)$.
   a) Draw a Voronoi diagram for the three pizza shops.
   b) If Arthur is at coordinates P(6,8), determine which store is closest to him for buying pizza.

5. In the Voronoi diagram shown below, R, S, and T represent the locations of the food storage facilities for each village. A fourth food storage facility will be constructed in the region that currently does not have one.



   a) Explain why the fourth food storage facility should be constructed closest to Village A.
   b) The segment of PQ has equation $y = 6$. If T located at $(8,9)$, find the value of the fourth food storage.

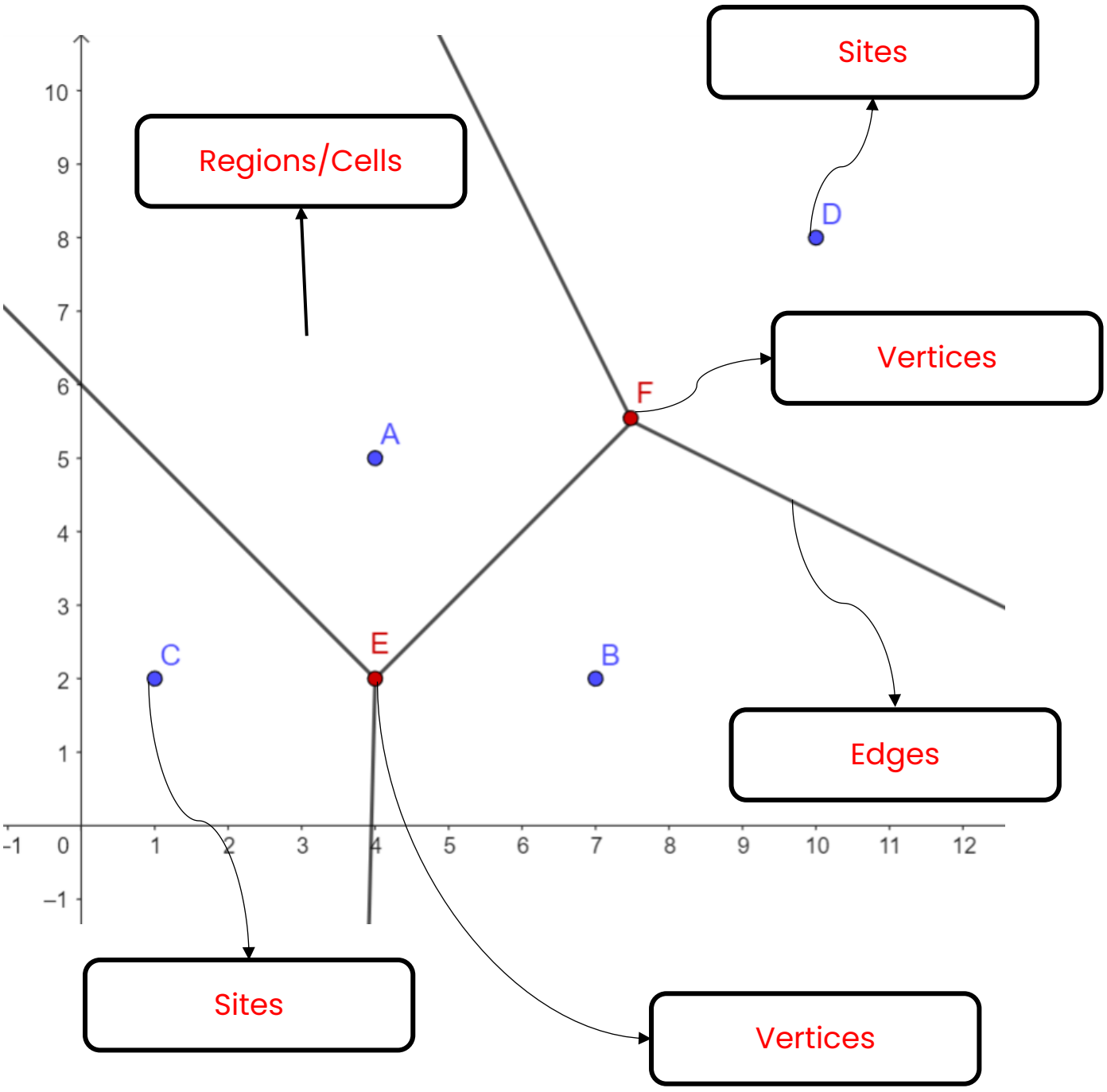
ANSWER KEY

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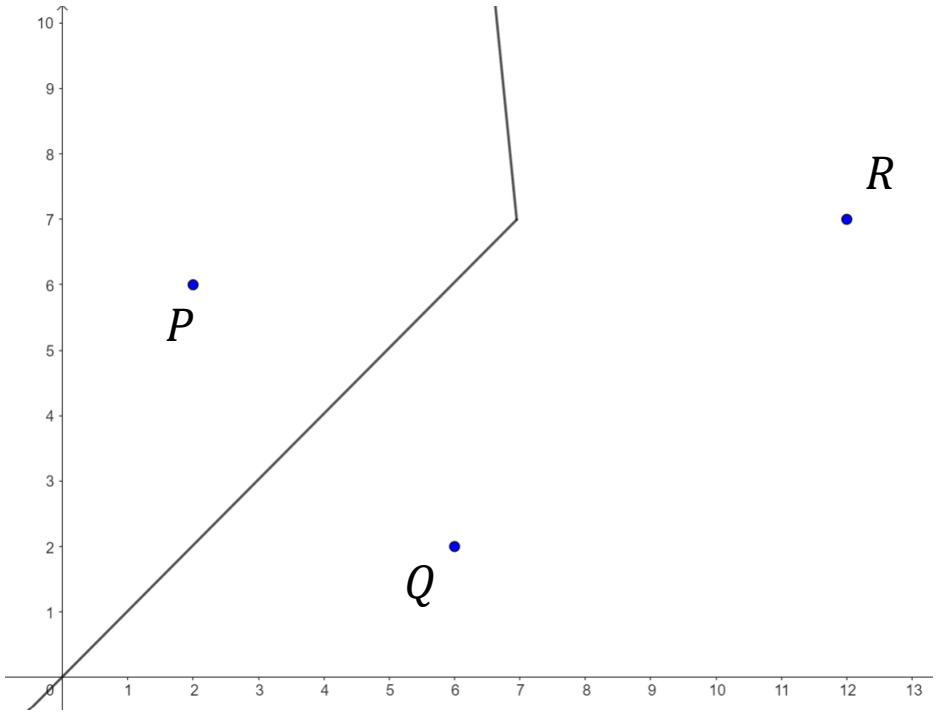
UNIT 3

Voronoi Diagrams

1. Write down the parts of the Voronoi diagram below.



2. A map of water towers in Sun City is shown in the figure below. Three towers, P, Q, and R, are located at the coordinates given.



a) Find the gradient of segment QR.

b) Find the equation needed to complete the Voronoi diagram (in the form of $y = mx + c$).

c) Draw the line to complete the diagram.

Answer:

a) $Q(6,2)$ and $R(12,7)$

$$m_{QR} = \frac{7-2}{12-6} = \frac{5}{6}$$

b) Each edge of a Voronoi diagram is the perpendicular bisector of the line segment connecting two sites.

$$m_{QR} = m_1 = \frac{5}{6}$$

$$m_1 \times m_2 = -1$$

$$\frac{5}{6} \times m_2 = -1$$

$$m_2 = -\frac{6}{5}$$

$$m_2 = -1.2$$

$$mid_{QR} = \left(\frac{6+12}{2}, \frac{2+7}{2}\right)$$

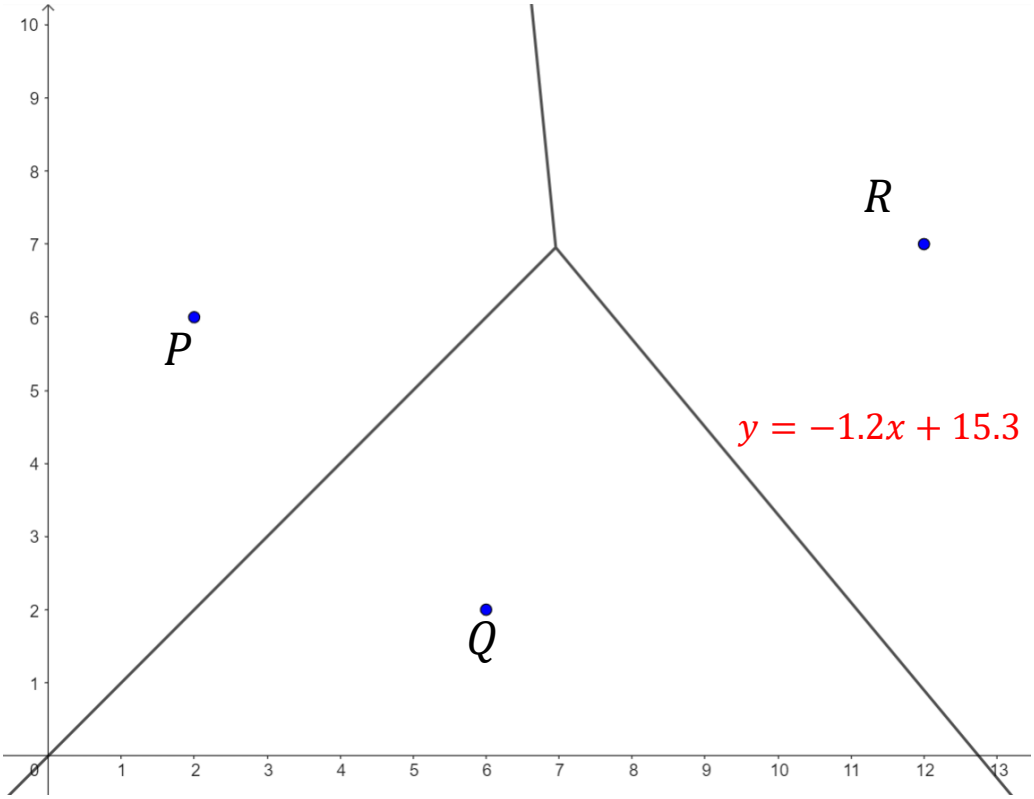
$$mid_{QR} = (9\ , 4.5\ )$$

$$y - 4.5 = -1.2(x - 9)$$

$$y = -1.2x + 10.8 + 4.5$$

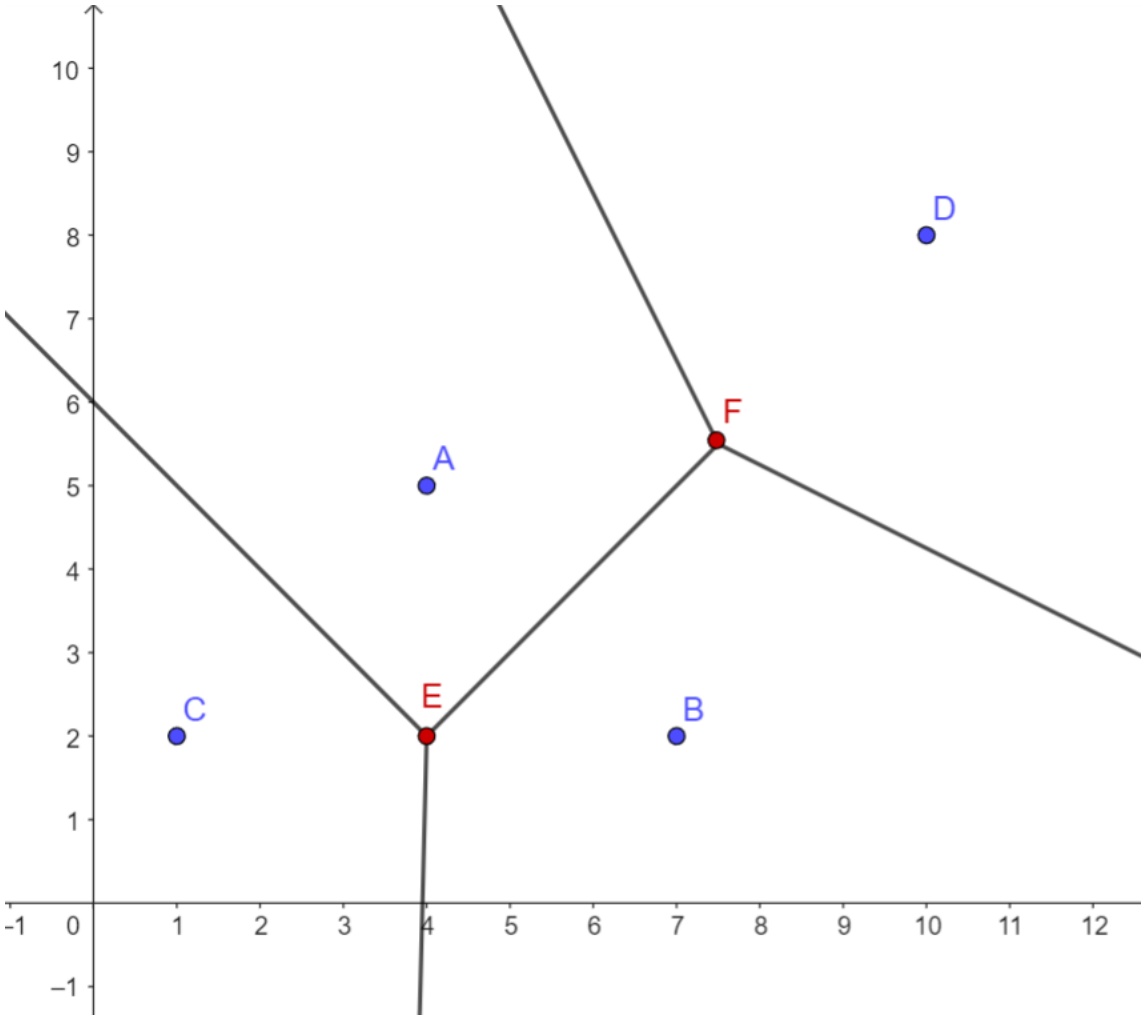
$$y = -1.2x + 15.3$$

c) The complete diagram.



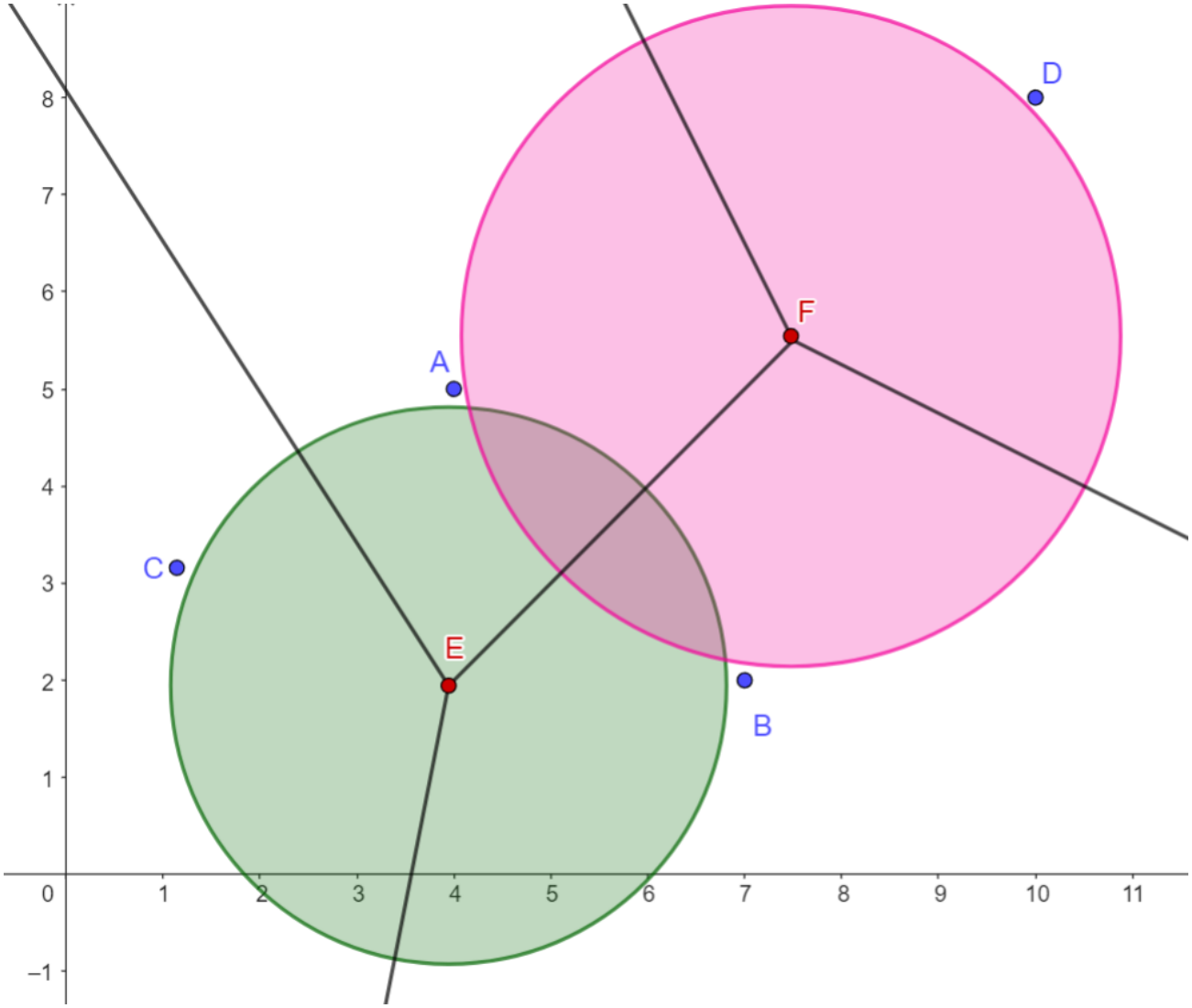
The line $y = -1.2\,x + 15.3$ is drawn without intersecting the vertices, so as not to create any additional regions beyond those already present.

3. In the game *Clash of Kings*, players require not only attacking skills but also effective strategies for building defences. Thomas is working on constructing an archer tower as part of his defence strategy. The defence plan he developed is illustrated in the diagram below, where points A, B, C, and D represent various barbarian camps. Determine the optimal location for the tower to achieve the maximum shooting range, using the concept from the toxic waste dump problem. Explain your reasoning, and include a circle in the provided plan to represent the archer's shooting range.



Answer:

The toxic waste dump problem models the restricted area as a circle, with the centre of the circle located on one of the vertices. No one is permitted to enter this circular region. In this case, a decision must be made regarding the location of the archer tower.



Of the two circles, the pink one is larger than the green one. Therefore, the archer tower should be built at point F, as it provides a greater shooting range than point E.

4. Two pizza shops are located at coordinates $A(2,4)$ and $B(7,2)$. The plan is to build a third shop at coordinates $C(10,7)$.

a) Draw a Voronoi diagram for the three pizza shops.

b) If Arthur is at coordinates P(6,8), determine which store is closest to him for buying pizza.

Answer:

a) The edges of a Voronoi diagram are formed using perpendicular bisectors. Therefore, we will find the perpendicular bisectors of segments AB, AC, and BC.

- Gradient.

$$m_{AB} = m_1 = \frac{2-4}{7-2} = -\frac{2}{5} \qquad m_1 \times m_4 = -1 \rightarrow m_4 = \frac{5}{2}$$

$$m_{AC} = m_2 = \frac{7-4}{10-2} = \frac{3}{8} \qquad m_2 \times m_5 = -1 \rightarrow m_5 = -\frac{8}{3}$$

$$m_{BC} = m_3 = \frac{7-2}{10-7} = \frac{5}{3} \qquad m_3 \times m_6 = -1 \rightarrow m_6 = -\frac{3}{5}$$

- Midpoints.

$$mid_{AB} = \left(\frac{2+7}{2}, \frac{4+2}{2}\right) = (4.5\,, 3)$$

$$mid_{AC} = \left(\frac{10+2}{2}, \frac{7+4}{2}\right) = (6\,, 5.5)$$

$$mid_{BC} = \left(\frac{7+10}{2}, \frac{2+7}{2}\right) = (8.5\,, 4.5)$$

- Perpendicular equations:

Perpendicular to AB

$$y - 3 = \frac{5}{2}(x - 4.5)$$

$$y = \frac{5}{2}x - \frac{22.5}{2} + 3$$

$$y = 2.5x - 8.25$$

Perpendicular to AC

$$y - 5.5 = -\frac{8}{3}(x - 6)$$

$$y = -\frac{8}{3}x + 16 + 5.5$$

$$y = -\frac{8}{3}x + 21.5$$

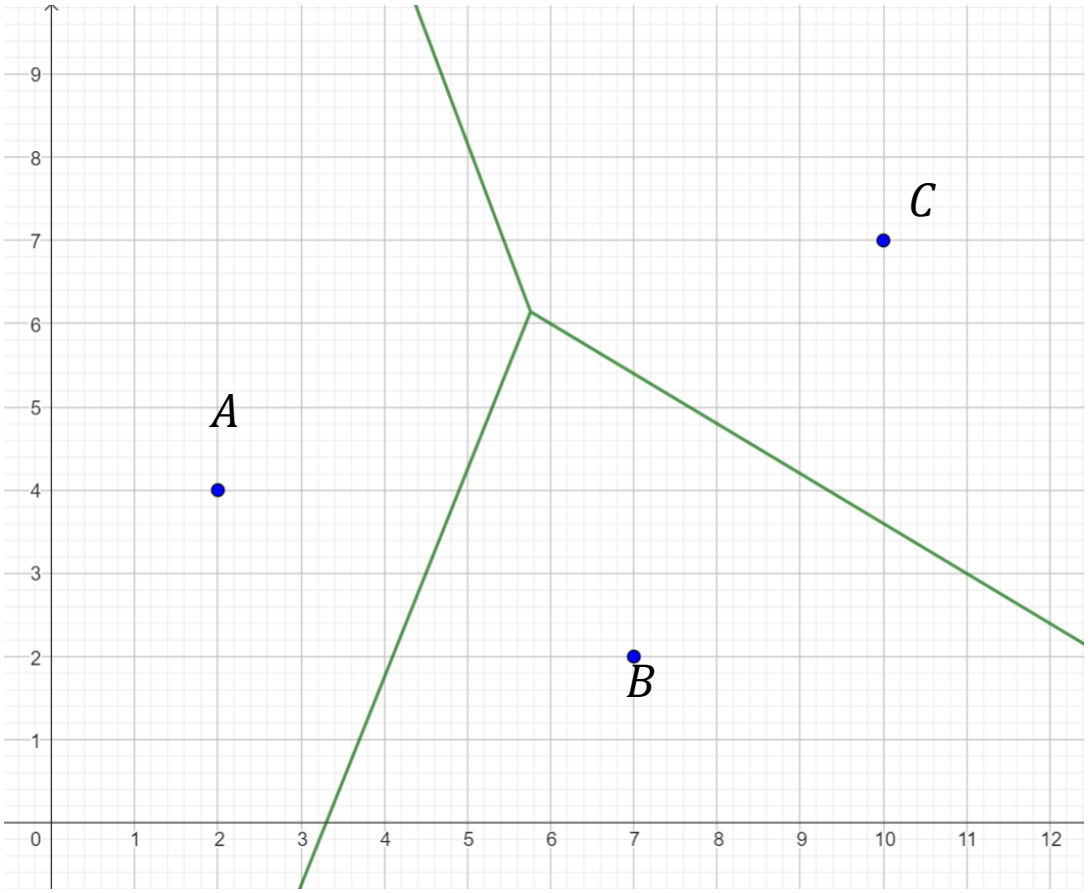
Perpendicular to BC

$$y - 4.5 = -\frac{3}{5}(x - 8.5)$$

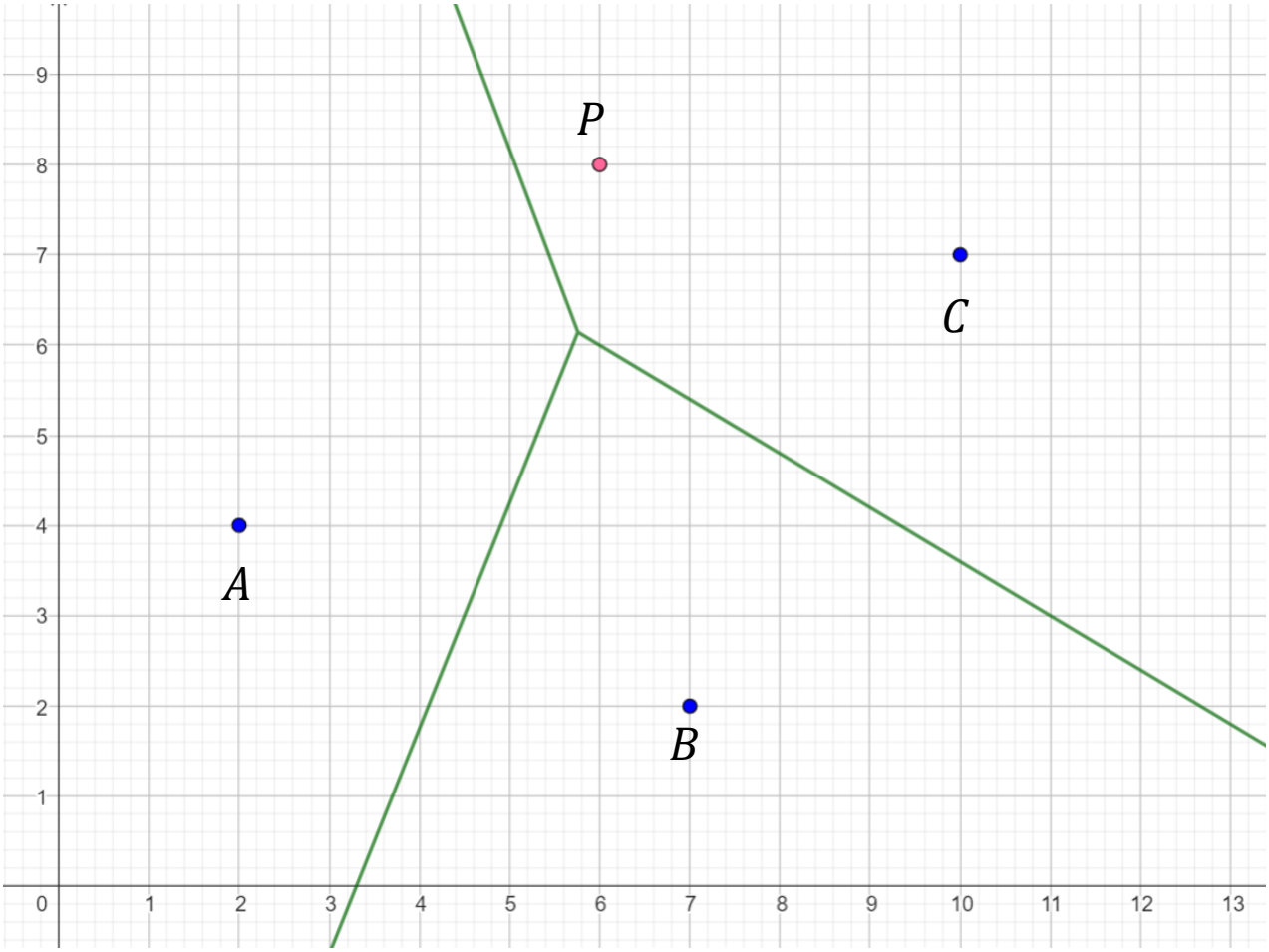
$$y = -\frac{3}{5}x + \frac{25.5}{5} + 4.5$$

$$y = -\frac{3}{5}x + 9.6$$

The intersection of the three lines forms the vertices. Remove any extraneous lines to complete the Voronoi diagram as shown.

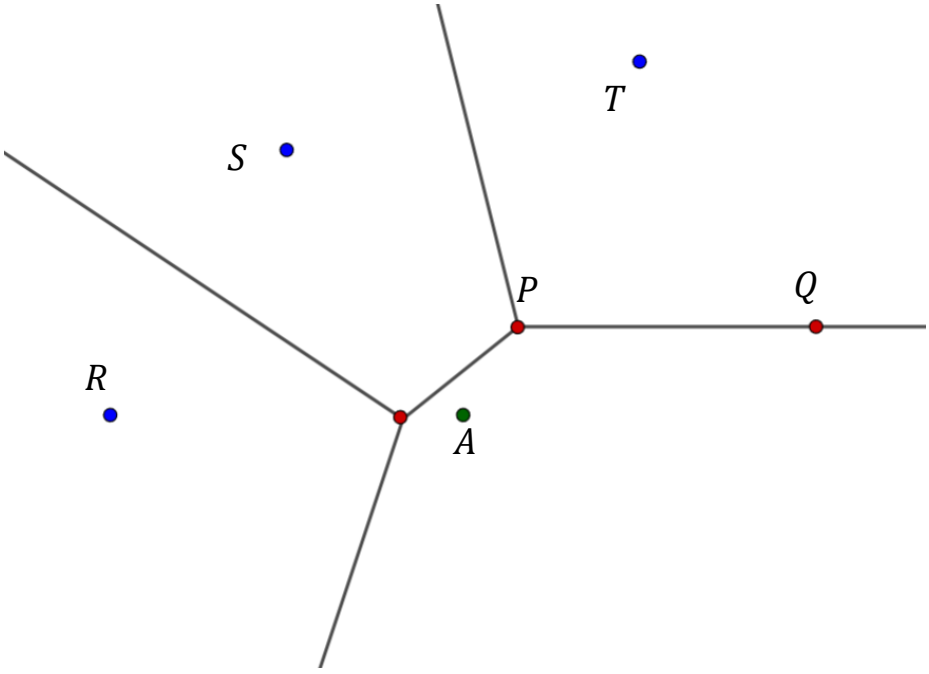


b)



The closest pizza shop to Arthur is Shop C, as Arthur is located within the region corresponding to Shop C.

5. In the Voronoi diagram shown below, R, S, and T represent the locations of the food storage facilities for each village. A fourth food storage facility will be constructed in the region that currently does not have one.

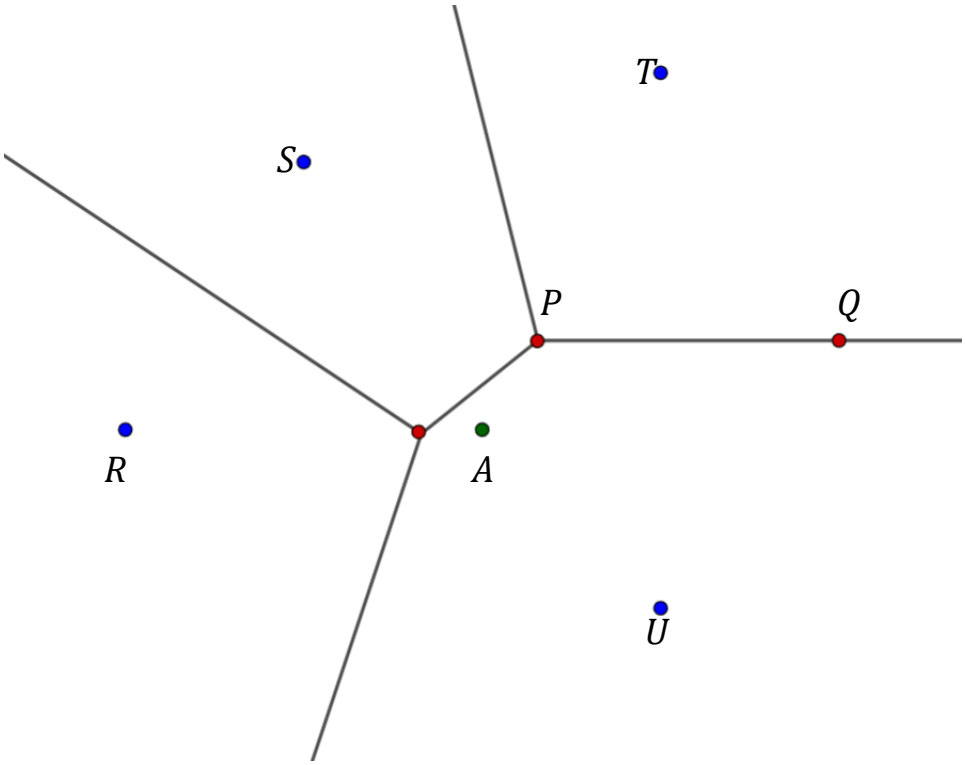


a) Explain why the fourth food storage facility should be constructed closest to Village A.

b) The segment of PQ has equation $y = 6$. If T located at $(8,9)$, find the value of the fourth food storage.

Answer:

a) The fourth food storage facility should be constructed closest to Village A, as it currently lacks coverage according to the Voronoi diagram.



b) PQ is the perpendicular between T and U. Since PQ is the perpendicular bisector of the segment connecting T and U, and PQ lies along the line $y = 6$, T and U must be mirror images of each other across this line.

Therefore, the coordinates that are equidistant from T at PQ are.

$$U = (8, 2 \times 6 - 9) = (8,3)$$