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**UNIT 5**

# SL 5.7 — Optimisation in Context

Objective function • Constraints • Critical points • First derivative test • Endpoints • Interpret in context

**A. OPTIMISATION TOOLKIT (FILL IN THE BLANKS)**

Use short phrases. Think: variables → function → calculus → check → interpret.

1. **Define variables** and write the **objective function**:

   _____

2. **State constraints** and the allowed domain:

   _____

3. Find the **first derivative** and solve the **first-order condition** (FOC):

   _____

4. **Test critical point(s)** (and **endpoints** if included):

   _____

5. Answer the question **in context** (include units + what the value means):

   _____

**B. GUIDED EXAMPLE 1 — OPEN-TOP BOX (MAXIMUM VOLUME)**

A 20 cm × 20 cm square sheet is used. Equal squares of side length **x** are cut from the corners and the sides are folded up.

1. Let **x** be the cut-out size (cm). Write the base dimensions and height:

   Base: ( _____ ) cm by ( _____ ) cm, Height: _____ cm

2. Write the volume function:

   $V(x) =$ _____, domain: _____

3. Differentiate and solve the FOC (calculator algebra allowed):

   $V'(x) = 0 \Longrightarrow x \approx$ _____ cm

4. State the answer in context (what should be cut out to maximise volume?):

   Cut out squares of side length _____ cm.

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### C. GUIDED EXAMPLE 2 — PENS (MAXIMUM PROFIT)

The demand function is $p(x) = 10 - 0.5x$. Here **x** is the number of pens sold (in hundreds). Each pen costs $2 to produce.

1. Write revenue and cost (in dollars) in terms of **x**:

   Revenue: $R(x) = 100xp(x)$ = _____

   Cost: _____

2. Write the profit function and simplify:

   $P(x) = 100xp(x) - C(x) - 200x$ = $P(x) = 800x - 50x^2$

3. Maximise profit using calculus:

   $P'(x) = 800 - 100x$ = 0 $\Longrightarrow$ x = _____

4. Answer in context:

   Sell _____ hundred pens = _____ pens to maximise profit.

### D. GUIDED EXAMPLE 3 — GARDEN (MINIMUM COST)

A rectangular garden has area 100 m². One side is along a wall (no fence needed). Fencing costs $10 per meter.

1. Let width **w** be perpendicular to the wall and length **l** be along the wall. Write the constraint:

   $wl = 100$ $\Longrightarrow$ l = _____

2. Write the fence length and the cost function in terms of **w**:

   Fence length = _____

   Cost: $C(w) = 10(2w + l)$

3. Minimise cost:

   $C'(w) = 10(2 - \frac{100}{w^2})$ = 0 $\Longrightarrow$ w = _____ m

4. Answer in context:

   Width ≈ _____ m, Length ≈ _____ m.

**E. EXTENSION PROBLEMS (EXAM-STYLE)**

Show your working clearly. Use calculus and interpret the solution.

1. **Packaging (open-top box).** An open-top box is made by cutting out squares of side **x** from the corners of a 30 cm × 18 cm rectangle and folding up the sides.

   (a) Write a volume function V(x) and its domain. (b) Find the value of x that maximises volume.

2. **Economics (price + advertising).** A product has demand (units) **q(p)=1200-30p**, where **p** is the price (dollars). If the unit cost is $8 and a fixed advertising campaign costs $1500,

   (a) Write a profit function Π(p). (b) Find the price that maximises profit and the maximum profit.

3. **Geometry (minimum perimeter).** A rectangle has fixed area 200 m².

   (a) Write perimeter P in terms of one variable. (b) Find the dimensions that minimise P.

# ANSWER KEY

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**ANSWERS — GUIDED EXAMPLES**

1. **B.** Base: $(20-2x)$ by $(20-2x)$, height $x$. Volume: $V(x)=x(20-2x)^2$ with domain $0<x<10$.

   Solve $V'(x)=0$. Maximum occurs at $x \approx 2.89$ cm (GDC), so cut out 2.89 cm squares.

2. **C.** Revenue $R(x)=100x(10-0.5x)$. Cost $C(x)=200x$. Profit: $P(x)=-50x^2+800x-200$

   so $P'(x)=800-100x=0 \Rightarrow x=8$. Sell 800 pens.

3. **D.** Constraint $wl=100 \Rightarrow l=100/w$. Fence length $= 2w+l = 2w+100/w$.

   Minimise $C(w)=10(2w+100/w)$. Set $C'(w)=0 \Rightarrow 2-100/w^2=0 \Rightarrow w=\sqrt{50}\approx 7.07$, $l\approx 14.14$.

**ANSWERS — EXTENSION PROBLEMS**

1. **E1.** $V(x)=x(30-2x)(18-2x)$, domain $0<x<9$. Max occurs at $x \approx 3.87$ cm (GDC / derivative).

2. **E2.** Profit $\Pi(p)=(p-8)(1200-30p)-1500 = -30p^2+1440p-11100$.

   Vertex at $p = -b/(2a)=1440/(60)=24$. Maximum profit $\Pi(24)=\$6{,}180$.

3. **E3.** If sides are $x$ and $200/x$, then $P(x)=2x+400/x$. Min at $x=\sqrt{200}\approx 14.14$, other side $\approx 14.14$.

   A square gives the minimum perimeter: 14.14 m by 14.14 m.