Name: _____ Date: _____

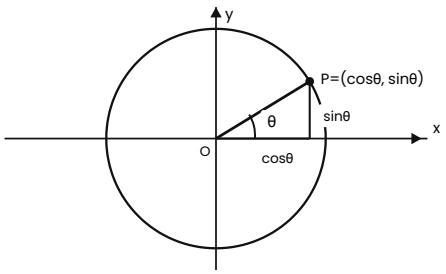
Unit 3

SL 3.5 — Trigonometry and the Unit Circle

Unit circle • Exact values • Quadrant signs • Angle relationships • Lines through the origin • Ambiguous sine rule

A. UNIT CIRCLE DEFINITIONS

The circle has centre O and radius 1. Complete the statements using the coordinates of point P.



1. **The point P on the unit circle has coordinates P = (** _____ **,** _____ **).**
2. **On the unit circle,** $\sin\theta = y$, $\cos\theta = x$**. Therefore, sine is the** _____ **-coordinate and cosine is the** _____ **-coordinate.**
3. **Since the radius is 1, the hypotenuse of the right triangle is** _____ **.**
4. **Complete:** $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{y}{x}$**. Tangent is also the** _____ **of the radius/line.**

B. DEGREES, RADIANS, AND EXACT VALUES

Use conversion factors and the special triangles from the lesson.

1. **Convert** 210° **to radians.**

2. **Convert** $5\pi/6$ **radians to degrees.**

3. **Write the two conversion formulas:** $\theta_{\text{rad}} = \theta^\circ \cdot \frac{\pi}{180}$ **and** $\theta^\circ = \theta_{\text{rad}} \cdot \frac{180}{\pi}$.

θ	sin(θ)	cos(θ)	tan(θ)
0			
30° = π/6			
45° = π/4			
60° = π/3			
90° = π/2			

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Unit 3

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C. QUADRANT SIGNS AND REFERENCE ANGLES

Remember: All Students Take Calculus.

Quadrant	sin(θ)	cos(θ)	tan(θ)	Reason from coordinates
I				x and y are both positive.
II				x is negative and y is positive.
III				x and y are both negative.
IV				x is positive and y is negative.

1. Find the exact values: (a) $\sin(5\pi/6)$ _____ **(b)** $\cos(7\pi/6)$ _____ **(c)** $\tan(11\pi/6)$ _____

2. If $\sin\theta = 3/5$ **and** $\pi/2 < \theta < \pi$**, find** $\cos\theta$ **and** $\tan\theta$**.**

D. RELATIONSHIPS BETWEEN ANGLES

Use reflections and rotations on the unit circle.

1. Complete: $\cos(-\theta) =$ _____ **because the x-coordinate** _____ .

2. Complete: $\sin(-\theta) =$ _____ **because the y-coordinate** _____ .

3. Complete: $\cos(\pi - \theta) =$ _____ **and** $\sin(\pi - \theta) =$ _____ .

4. Use a relationship to find $\cos(2\pi/3)$ **exactly.**

5. Use the unit circle to explain why $\tan(\theta + \pi) = \tan\theta$**.**

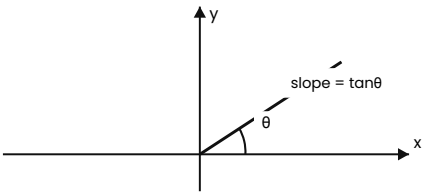
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Unit 3

SL 3.5 — Trigonometry and the Unit Circle

E. LINES THROUGH THE ORIGIN

For a line through the origin making a positive angle θ with the x-axis, the gradient is tan(θ), so $y = (\tan\theta)x$.



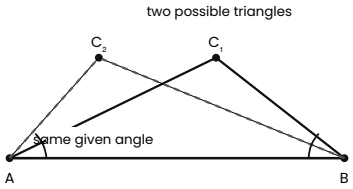
1. **Find the equation of the line through the origin making an angle of $\pi/6$ with the positive x-axis.**

2. **The line L passes through the origin and makes an angle of 150° with the positive x-axis.**
   (a) Find the gradient of L. _____
   (b) Write the equation of L. _____
   (c) Find the acute angle between L and the x-axis.
   _____

3. **A line through the origin intersects the unit circle at a point P in quadrant III. If its gradient is $\sqrt{3}$, find one possible value of θ in radians and the coordinates of P.**

F. AMBIGUOUS CASE OF THE SINE RULE

When two sides and a non-included angle are known, two triangles may be possible. Start with the sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.



1. **In triangle ABC, $A = 35^\circ$, $a = 7$, and $b = 10$. Find the possible values of B, C, and c.**

2. **Explain how you decide whether the obtuse value of B is valid.**

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Unit 3

SL 3.5 — Trigonometry and the Unit Circle

G. EXTENDED THINKING

Show clear reasoning. These are designed to connect exact values, unit-circle symmetry, and modelling.

1. **A point P on the unit circle lies in quadrant II and has y-coordinate $\sqrt{7}/4$.**
   (a) Find the x-coordinate of P exactly.
   (b) Find $\tan\theta$ exactly.
   (c) Find $\sin(\theta+\pi)$ and $\cos(-\theta)$.

2. **The line $y = mx$ intersects the unit circle at a point in quadrant IV. The angle measured anticlockwise from the positive x-axis is $\theta = 11\pi/6$.**
   (a) Find m exactly. (b) Find the coordinates of the point of intersection. (c) State another angle in $[0, 2\pi)$ with the same tangent value.

3. **In triangle ABC, $A = 42^\circ$, $a = 8.4$, and $b = 10.1$.**
   (a) Find all possible values of B and C.
   (b) Find the two possible lengths of c.
   (c) Explain why a calculator answer for $\sin^{-1}(x)$ does not automatically give every triangle.

ANSWER KEY

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Unit 3

SL 3.5 — Solutions

A. UNIT CIRCLE DEFINITIONS

1. P = $(\cos\theta, \sin\theta)$.
2. $\sin\theta = y$, $\cos\theta = x$. **Sine is the y-coordinate; cosine is the x-coordinate.**
3. **1**.
4. $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{y}{x}$. **Tangent is the gradient/slope.**

B. DEGREES, RADIANS, AND EXACT VALUES

1. $210^\circ \cdot \pi/180 = 7\pi/6$; answer: $7\pi/6$.
2. $(5\pi/6) \cdot 180/\pi = 150^\circ$; answer: **150°**.

θ	sin(θ)	cos(θ)	tan(θ)
0	**0**	**1**	**0**
30° = π/6	1/2	$\sqrt{3}/2$	$\sqrt{3}/3$
45° = π/4	$\sqrt{2}/2$	$\sqrt{2}/2$	**1**
60° = π/3	$\sqrt{3}/2$	1/2	$\sqrt{3}$
90° = π/2	**1**	**0**	**Undefined**

C. QUADRANT SIGNS AND REFERENCE ANGLES

Quadrant	sin(θ)	cos(θ)	tan(θ)
I	+	+	+
II	+	−	−
III	−	−	+
IV	−	+	−

1. (a) 1/2 (b) $-\sqrt{3}/2$ (c) $-\sqrt{3}/3$.
2. **Since θ is in QII,** $\cos\theta = -4/5$ **and** $\tan\theta = (3/5)/(-4/5) = -3/4$**.**

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Unit 3

SL 3.5 — Solutions

D. RELATIONSHIPS BETWEEN ANGLES

1. $\cos(-\theta) = \cos\theta$; **the x-coordinate does not change when reflected across the x-axis.**
2. $\sin(-\theta) = -\sin\theta$; **the y-coordinate changes sign when reflected across the x-axis.**
3. $\cos(\pi - \theta) = -\cos\theta$ and $\sin(\pi - \theta) = \sin\theta$.
4. $2\pi/3 = \pi - \pi/3$, so $\cos(2\pi/3) = -\cos(\pi/3) = -1/2$.
5. **Adding π moves the point to the opposite side of the circle, so both x and y change sign. The ratio y/x is unchanged, so tangent has period π.**

E. LINES THROUGH THE ORIGIN

1. $m = \tan(\pi/6) = \sqrt{3}/3$, so $y = (\sqrt{3}/3)x$.
2. (a) $\tan 150^\circ = -\sqrt{3}/3$. (b) $y = (-\sqrt{3}/3)x$. (c) **30°**.
3. **In QIII, the reference angle is $\pi/3$, so $\theta = 4\pi/3$. The point is $(-1/2, -\sqrt{3}/2)$.**

F. AMBIGUOUS CASE OF THE SINE RULE

1. $\sin B/10 = \sin 35^\circ/7$, so $\sin B \approx 0.8194$. Thus **B ≈ 55.0° or 125.0°**.
   If $B \approx 55.0^\circ$, then **C ≈ 90.0°** and $c = 7\sin 90^\circ/\sin 35^\circ \approx 12.2$.
   If $B \approx 125.0^\circ$, then **C ≈ 20.0°** and $c = 7\sin 20^\circ/\sin 35^\circ \approx 4.17$.
2. **Check whether A plus the obtuse angle is less than 180°. If the angle sum reaches or exceeds 180°, the obtuse case is invalid.**

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Unit 3

SL 3.5 — Solutions

G. EXTENDED THINKING

1. $x^2 + (\sqrt{7}/4)^2 = 1$, so $x^2 = 9/16$. Since P is in QII, $x = -3/4$.
   $\tan\theta = (\sqrt{7}/4)/(-3/4) = -\sqrt{7}/3$.
   $\sin(\theta + \pi) = -\sin\theta = -\sqrt{7}/4$ and $\cos(-\theta) = \cos\theta = -3/4$.

2. $m = \tan(11\pi/6) = -\sqrt{3}/3$.
   The unit-circle point is $(\sqrt{3}/2, -1/2)$.
   Another angle in $[0, 2\pi)$ with the same tangent value is $5\pi/6$.

3. $\sin B/10.1 = \sin 42^\circ/8.4$, so $\sin B \approx 0.8045$.
   B ≈ 53.6° or 126.4°. Both are valid because $42^\circ + 126.4^\circ < 180^\circ$.
   If $B \approx 53.6^\circ$, then **C ≈ 84.4°** and $c \approx 12.5$.
   If $B \approx 126.4^\circ$, then **C ≈ 11.6°** and $c \approx 2.52$.
   A calculator's inverse sine usually returns the acute angle only, so the supplementary angle must be checked separately.