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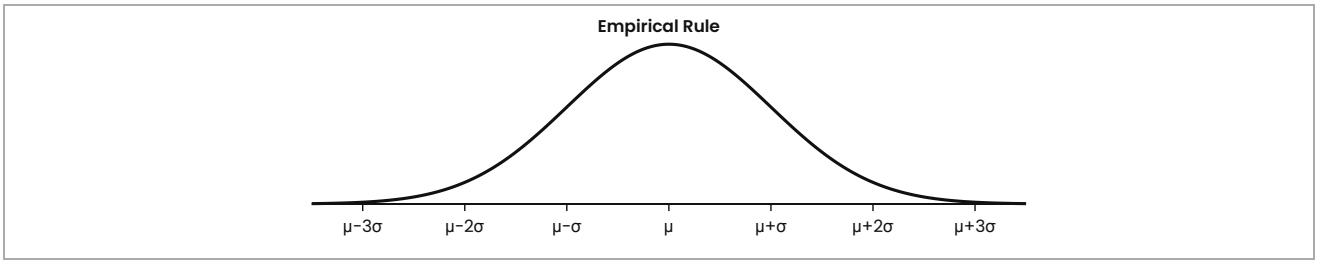
SL 4.9 — The Normal Distribution

Bell Curves • Empirical Rule • Normal CDF • Inverse Normal • Modelling

A. KEY IDEAS: NORMAL DISTRIBUTIONS

Fill in each blank. Use precise language: continuous, symmetrical, mean, standard deviation, variance.

1. **A normal distribution is a _____ probability distribution with a _____ and symmetrical shape.**
2. **In $X \sim N(\mu, \sigma^2)$, the mean is _____, the standard deviation is _____, and the variance is _____.**
3. **The total area under a normal curve is _____, and the area to the left of the mean is _____.**
4. **For a continuous normal variable, $P(X = c) = 0$ because a single point has _____.**
5. **The Empirical Rule says approximately _____ % lies within one SD, _____ % within two SDs, and _____ % within three SDs.**



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SL 4.9 — The Normal Distribution

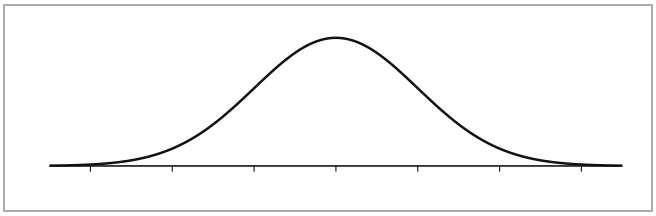
Bell Curves • Empirical Rule • Normal CDF • Inverse Normal • Modelling

B. BELL CURVE AND EMPIRICAL RULE PRACTICE

A set of IQ scores is normally distributed with mean 100 and standard deviation 15.

1. **Write the distribution using normal notation:** _____

2. **Draw and label the curve at the mean, one SD below/above, and two SDs above.**



3. **Estimate the percentage of scores between 85 and 115 using the Empirical Rule.**

_____

4. **Estimate the percentage of scores above 130.**

_____

5. **Explain why $P(X = 100) = 0$ even though 100 is the mean.**

_____

C. COMPARING NORMAL CURVES

Use the slide idea: same variance/different mean and same mean/different variance.

Distribution	Mean	SD	What changes on the graph?
$A \sim \mathrm{N}(20, 3^2)$			
$B \sim \mathrm{N}(28, 3^2)$			
$C \sim \mathrm{N}(20, 8^2)$			

1. **Which two distributions have the same variance but different means?**

_____

2. **Which two distributions have the same mean but different variance?**

_____

3. **Which curve is widest and lowest? Justify your answer.**

_____

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D. NORMAL CDF CALCULATIONS

A manufacturer produces bolts whose lengths are normally distributed with mean 8.0 cm and standard deviation 0.1 cm. Let $X \sim \mathrm{N}(8.0, 0.1^2)$.

1. **Find the probability that a randomly selected bolt is shorter than 7.9 cm.**

2. **Find the probability that a bolt is between 7.9 cm and 8.1 cm.**

3. **Find the probability that a bolt is longer than 8.18 cm.**

4. **A batch contains 600 bolts. Estimate the number shorter than 7.85 cm.**

E. STANDARDISING BEFORE USING THE GDC

A runner's 100 m training time, in seconds, is modelled by $X \sim \mathrm{N}(12.4, 0.8^2)$.

1. **Standardise 11.2 seconds using $z = \frac{x - \mu}{\sigma}$, then find $P(X < 11.2)$.**

2. **Find $P(12.0 < X < 13.5)$.**

3. **A time of 14.0 seconds has what z-score? Interpret whether this is unusually slow.**

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F. INVERSE NORMAL CALCULATIONS

Heights of adult women are normally distributed with mean 165 cm and standard deviation 6 cm. Let $X \sim \mathrm{N}(165, 6^2)$.

1. **Find the height that marks the top 10% of women.**

2. **Find the height that marks the shortest 15% of women.**

3. **Find the interval containing the middle 80% of women's heights.**

4. **Explain why inverse normal uses a left-tail area, even when the question says "top 10%."**

_____

G. QUALITY CONTROL: WARRANTY THRESHOLDS

Battery life, in hours, is modelled by $X \sim \mathrm{N}(42, 5^2)$.

1. **A company wants only 3% of batteries to fail before the warranty cutoff. Find the cutoff time.**

2. **Find the probability a battery lasts between 35 and 45 hours.**

3. **In a shipment of 2000 batteries, estimate the number that last less than 35 hours.**

4. **Would a warranty cutoff of 35 hours be stricter or more generous than the cutoff from part (a)? Explain.**

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Unit 4

SL 4.9 — The Normal Distribution

Bell Curves • Empirical Rule • Normal CDF • Inverse Normal • Modelling

H. EXTENDED PROBLEM: FIND THE MEAN AND STANDARD DEVIATION

A filling machine's volume, in mL, is normally distributed. 5% of bottles contain less than 238 mL, and 10% contain more than 254 mL.

1. **Let $X \sim \mathrm{N}(\mu, \sigma^2)$. Write two equations using z-values and $x = \mu + z\sigma$.**

2. **Find μ and σ.**

H. EXTENDED PROBLEM CONTINUED

3. **Using your values of μ and σ, find the probability a bottle contains between 240 mL and 250 mL.**

4. **The machine is acceptable if fewer than 2% of bottles contain less than 235 mL. Should the machine be adjusted? Justify your answer.**

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Unit 4

SL 4.9 — The Normal Distribution

Bell Curves • Empirical Rule • Normal CDF • Inverse Normal • Modelling

I. EXTENDED PROBLEM: COMPARING TWO NORMAL MODELS

Two machines produce exam-style score estimates out of 100: $A \sim \mathrm{N}(80, 6^2)$ and $B \sim \mathrm{N}(78, 4^2)$.

1. **Which machine is more likely to produce a score above 85? Calculate both probabilities.**

2. **Which machine is more likely to produce a score between 70 and 90? Calculate both probabilities.**

3. **Find the 95th percentile for each machine.**

4. **Write one sentence comparing "high-end performance" and "consistency."**

J. EXTENDED PROBLEM: GRADE BOUNDARIES AND CONDITIONAL PROBABILITY

An exam score is modelled by $X \sim \mathrm{N}(62, 12^2)$. A grade 7 is awarded to the top 8% of students. A grade 6 or higher is awarded to the top 22%.

1. **Find the minimum score for a grade 7.**

2. **Find the minimum score for a grade 6.**

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Unit 4

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Bell Curves • Empirical Rule • Normal CDF • Inverse Normal • Modelling

J. EXTENDED PROBLEM CONTINUED

3. **The school considers a score above 50 to be a pass. Find the expected number of passing students in a cohort of 180.**

4. **Given that a randomly selected student passed, find the probability that the student scored above 70.**

5. **Find the score x such that exactly 40% of all students score between 50 and x.**

K. FINAL REFLECTION

1. **When should you use normal CDF rather than inverse normal?**

_____

2. **Why does the second parameter in $X \sim \mathrm{N}(\mu, \sigma^2)$ use σ^2 rather than σ?**

_____

3. **What calculator/GDC bounds would you use to approximate $P(X > k)$?**

_____

ANSWER KEY

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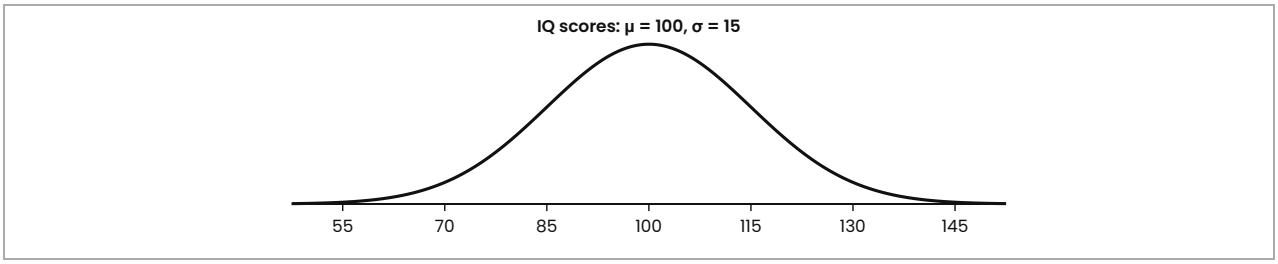
SL 4.9 — The Normal Distribution

A. KEY IDEAS: NORMAL DISTRIBUTIONS

1. **continuous; bell-shaped.**
2. **Mean: μ; standard deviation: σ; variance: σ^2.**
3. **Total area = 1; area to the left of the mean = 0.5.**
4. **A single point has zero width, so its area/probability is 0.**
5. **68%, 95%, 99.7%.**

B. BELL CURVE AND EMPIRICAL RULE PRACTICE

1. $X \sim \mathrm{N}(100, 15^2)$
2. **Curve should be centred at 100, with 85 and 115 one SD from the mean, and 130 two SDs above the mean.**



3. **About 68%; exact normal CDF gives $P(85 < X < 115) \approx 0.6827$.**
4. **About 2.5% by the Empirical Rule; exact normal CDF gives $P(X > 130) \approx 0.0228$.**
5. **Because 100 is only one exact value in a continuous distribution; probability comes from area over an interval.**

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Unit 4

SL 4.9 — The Normal Distribution

C. COMPARING NORMAL CURVES

1. **A and B have the same variance:** $3^2 = 9$.
2. **A and C have the same mean:** $\mu = 20$.
3. **C is widest and lowest because it has the largest standard deviation:** $\sigma = 8$.

D. NORMAL CDF CALCULATIONS

1. $P(X < 7.9) = 0.1587$.
2. $P(7.9 < X < 8.1) = 0.6827$. **This matches the one-standard-deviation Empirical Rule estimate.**
3. $P(X > 8.18) = 1 - P(X < 8.18) = 0.0359$.
4. $P(X < 7.85) = 0.0668$, **so** $600(0.0668) = 40.1$. **About 40 bolts.**

E. STANDARDISING BEFORE USING THE GDC

1. $z = \frac{11.2 - 12.4}{0.8} = -1.5$, **so** $P(X < 11.2) = 0.0668$.
2. $P(12.0 < X < 13.5) = 0.6069$.
3. $z = \frac{14.0 - 12.4}{0.8} = 2$. **This is unusually slow because it is two standard deviations above the mean; only about** 2.28% **are slower.**

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SL 4.9 — The Normal Distribution

F. INVERSE NORMAL CALCULATIONS

1. **Top 10% means left-tail area 0.90:** $\text{invNorm}(0.90, 165, 6) = 172.7$ **. Taller than 172.7 cm.**
2. $\text{invNorm}(0.15, 165, 6) = 158.8$ **. Shortest 15% are below about 158.8 cm.**
3. **Middle 80% leaves 10% in each tail:** $157.3 < x < 172.7$ **cm.**
4. **Inverse normal returns the x-value for a cumulative area to the left, so "top 10%" is entered as 0.90 to the left.**

G. QUALITY CONTROL: WARRANTY THRESHOLDS

1. **Use left-tail area 0.03:** $\text{invNorm}(0.03, 42, 5) = 32.6$ **. Cutoff ≈ 32.6 hours.**
2. $P(35 < X < 45) = 0.645$ **.**
3. $P(X < 35) = 0.0808$ **;** $2000(0.0808) = 161.5$ **. About 162 batteries.**
4. **A 35-hour cutoff is more generous to customers/stricter for the company, because about 8.08% fail before 35 hours, not just 3%.**

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Unit 4

SL 4.9 — The Normal Distribution

H. EXTENDED PROBLEM: FIND THE MEAN AND STANDARD DEVIATION

1. $P(X < 238) = 0.05$ **gives** $238 = \mu - 1.6449\sigma$; $P(X < 254) = 0.90$ **gives** $254 = \mu + 1.2816\sigma$.
2. **Subtract equations:** $16 = (1.2816 + 1.6449)\sigma$, **so** $\sigma = 5.47$ **mL and** $\mu = 247.0$ **mL.**
3. $P(240 < X < 250) = 0.608$.
4. $P(X < 235) = 0.0141 = 1.41\%$. **Since 1.41% is less than 2%, the machine does not need adjustment by this criterion.**

I. EXTENDED PROBLEM: COMPARING TWO NORMAL MODELS

1. $P(A > 85) = 0.202$; $P(B > 85) = 0.0401$. **Machine A is more likely to exceed 85.**
2. $P(70 < A < 90) = 0.904$; $P(70 < B < 90) = 0.976$. **Machine B is more likely to stay between 70 and 90.**
3. **A:** 89.9; **B:** 84.6.
4. **Machine A has stronger high-end performance, but Machine B is more consistent because it has the smaller standard deviation.**

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Unit 4

SL 4.9 — The Normal Distribution

J. EXTENDED PROBLEM: GRADE BOUNDARIES AND CONDITIONAL PROBABILITY

1. **Top 8% means left-tail area 0.92:** $\text{invNorm}(0.92, 62, 12) = 78.9$ **. Minimum grade 7 score ≈ 78.9.**
2. **Top 22% means left-tail area 0.78:** $\text{invNorm}(0.78, 62, 12) = 71.3$ **. Minimum grade 6 score ≈ 71.3.**
3. $P(X > 50) = 0.841$ **;** $180(0.841) = 151.4$ **. Expected passing students ≈ 151.**
4. $P(X > 70 \mid X > 50) = \frac{P(X > 70)}{P(X > 50)} = \frac{0.252}{0.841} = 0.300$ **. Probability ≈ 0.300.**
5. **Need** $P(50 < X < x) = 0.40$ **. Since** $P(X < 50) = 0.1587$ **, use left-tail area** $0.1587 + 0.40 = 0.5587$ **. Thus** $x = 63.8$ **.**

K. FINAL REFLECTION

1. **Use normal CDF when the x-value(s) are known and you need probability/area. Use inverse normal when the probability/area is known and you need the x-value.**
2. **The notation** $X \sim \text{N}(\mu, \sigma^2)$ **uses variance as the second parameter, and variance is the square of the standard deviation.**
3. **Use a very large upper bound, such as** 10^{99} **or the calculator's equivalent, for** $P(X > k)$ **.**