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Unit 1

SL 1.7 — Laws of Exponents and Logarithms

Rational Exponents • Logarithm Laws • Change of Base • Logarithmic & Exponential Equations

A. CORE IDEAS FROM THE LESSON

Complete each statement using precise mathematical language.

1. $a^{1/2}$ means the number which, when **squared**, gives _____, so $a^{1/2} =$ _____.
2. For rational exponents, the denominator gives the _____ and the numerator gives the _____.
3. Complete the general rule: $a^{n/m} =$ _____, where $a \geq 0$ in the real-number setting.
4. Explain why the base is often restricted to be non-negative for rational exponents with even roots: _____

B. RATIONAL EXPONENTS AND ROOTS

Rewrite first if helpful. Give exact answers.

1. Simplify $\frac{9^{5/2}}{3}$ = _____
2. Simplify $81^{-3/4}$ = _____
3. Simplify $(32^{2/5})^{3/2}$ = _____
4. Write $x^{7/3}$ in radical form: _____
5. Write $\sqrt[5]{y^3}$ using a rational exponent: _____
6. For $m > 0$, explain why $a^{n/m}$ and $(a^{1/m})^n$ are connected.

_____

C. ERROR ANALYSIS

Circle the error and correct it.

1. **A student writes $16^{3/4} = (16^3)^4$. The correct simplification is** _____.

2. **A student writes $(-4)^{1/2} = -2$ in the real-number system. What is wrong?**

_____

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D. EXPONENT LAWS AND LOGARITHM LAWS

Logarithm laws are inverse versions of exponent laws. Complete the missing entries.

Exponent law	Matching logarithm law	Condition
$x^m x^n = x^{m+n}$	$\log_a(xy) =$ _____	$a > 0,\ a \neq 1,\ x > 0,\ y > 0$
$\frac{x^m}{x^n} = x^{m-n}$	$\log_a\left(\frac{x}{y}\right) =$ _____	
$(x^m)^n = x^{mn}$	$\log_a(x^m) =$ _____	

E. CHANGE OF BASE

Use any valid base. In calculator work, base 10 or base e is common.

1. **Complete the formula:** $\log_a x =$ _____
2. **Express** $\log_5 45$ **in terms of** $\log 3$ **and** $\log 5$**:**

   _____
3. **Without using a calculator base button, write** $\log_2 7$ **using common logarithms:** _____
4. **Explain why** $\log_1 8$ **is undefined:** _____

F. PRACTICE: SOLVING LOGARITHMIC EQUATIONS

State the domain restrictions before solving. Reject extraneous solutions.

1. **Solve** $\log_2(x) + \log_2(x-2) = 3$**.** _____
2. **Solve** $\log_3(x+6) - \log_3(x-2) = 2$**.**

   _____
3. **Solve** $2\log_5(x) - \log_5(4) = 1$**.** _____

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G. MORE EQUATIONS AND DOMAINS

The logarithm argument must be positive.

1. **Solve** $\log_4(x-1)+\log_4(x+2)=1$.

2. **Solve** $\log_{10}(2x+1)=1+\log_{10}(x-4)$.

3. **For** $f(x)=\log_2(x+5)-\log_2(x-1$**, state the domain and solve** $f(x)=2$.

4. **Find the domain of** $g(x)=\log_3(12-x^2)$.

H. EXPONENTIAL EQUATIONS USING LOGARITHMS

Use change of base to solve exactly, then approximate when requested.

1. **Solve** $3^{x+1}=7$**. Give an exact answer and a decimal approximation.**

2. **A population model is** $P=500(1.07)^t$**. How long until the population reaches 1000?** _____

3. **A medicine amount is modeled by** $A=120(0.82)^t$**, where** t **is in hours. When is the amount 30 mg?** _____

I. COMMUNICATING WITH LOGS

Write each answer as a single expression.

1. **If** $\log_3 p=\alpha$ **and** $\log_3 q=\beta$**, express** $\log_3\left(\frac{p^2}{\sqrt{q}}\right)$ **in terms of** α **and** β.

2. **If** $\log_b 64=3$**, find** b. _____

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J. EXTENDED IB-STYLE APPLICATIONS

Show enough work to justify the model and the use of logarithms.

1. **The intensity ratio of two earthquakes is modeled by $\frac{I_2}{I_1} = 10^{M_2 - M_1}$. One earthquake is 250 times as intense as another.**

   (a) Write an equation for the magnitude difference $d = M_2 - M_1$. _____

   (b) Find d to 3 significant figures. _____

2. **A deposit grows from 800 dollars to 1200 dollars at 4.8% annual compound interest.**

   (a) Write an equation for t. _____

   (b) Solve for t to 2 decimal places. _____

3. **A sound level changes from L_1 to L_2 and $L_2 - L_1 = 10\log_{10}\left(\frac{I_2}{I_1}\right)$. If the intensity is multiplied by 8, find the increase in decibels to 1 decimal place.**

   _____

K. NON-ROUTINE REASONING

These questions are intended to be more difficult than routine substitution.

1. **Solve $\log_2(x-1) + \log_2(x+3) = \log_2(5x+3)$.**

   _____

2. **Find k if $\log_2(x) + \log_2(x-k) = 3$ has the solution $x = 4$.**

   _____

3. **For positive a and $a \neq 1$, prove that if $\log_a x = \log_a y$, then $x = y$.**

   _____

L. REFLECTION

1. **Which step most often creates extraneous solutions in logarithmic equations?** _____
2. **Write one original logarithmic equation that has exactly one valid solution.**

   _____

ANSWER KEY

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A. CORE IDEAS FROM THE LESSON

1. a; $\sqrt{a}$.
2. **The denominator gives the root; the numerator gives the power.**
3. $a^{n/m} = \sqrt[m]{a^n} = (\sqrt[m]{a})^n$.
4. **Even roots of negative numbers are not real, for example $(-1)^{1/2}$ is not a real number.**

B. RATIONAL EXPONENTS AND ROOTS

1. $\frac{9^{5/2}}{3} = \frac{(\sqrt{9})^5}{3} = \frac{3^5}{3} = 81$
2. $81^{-3/4} = \frac{1}{27}$
3. $(32^{2/5})^{3/2} = 8$
4. $x^{7/3} = \sqrt[3]{x^7} = x^2\sqrt[3]{x}$
5. $\sqrt[5]{y^3} = y^{3/5}$
6. **The denominator root is applied before raising to the numerator power, so the two forms are equivalent for the stated real-number conditions.**

C. ERROR ANALYSIS

1. $16^{3/4} = (\sqrt[4]{16})^3 = 2^3 = 8$.
2. $\sqrt{-4}$ **is not real;** -2 **squared gives** 4**, not** -4.

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D. EXPONENT LAWS AND LOGARITHM LAWS

Exponent law	Matching logarithm law	Condition
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$\frac{x^m}{x^n} = x^{m-n}$	$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$	
$(x^m)^n = x^{mn}$	$\log_a(x^m) = m\log_a x$	

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E. CHANGE OF BASE

1. $\log_a x = \frac{\log_b x}{\log_b a}$.
2. $\log_5 45 = \frac{\log(45)}{\log 5} = \frac{2\log 3 + \log 5}{\log 5}$.
3. $\log_2 7 = \frac{\log 7}{\log 2}$.
4. **Base 1 is invalid because** $1^y = 1$ **for all** y**, so it cannot produce 8.**

F. PRACTICE: SOLVING LOGARITHMIC EQUATIONS

1. **Domain** $x > 2$. $\log_2(x(x-2)) = 3$, **so** $x(x-2) = 8$. $x^2 - 2x - 8 = 0$. **Valid solution:** $x = 4$.
2. **Domain** $x > 2$. $\log_3\left(\frac{x+6}{x-2}\right) = 2$. **Hence** $\frac{x+6}{x-2} = 9$, **so** $x = 3$.
3. **Domain** $x > 0$. $\log_5\left(\frac{x^2}{4}\right) = 1$; $x^2 = 20$; **valid solution** $x = 2\sqrt{5}$.

G. MORE EQUATIONS AND DOMAINS

1. **Domain** $x > 1$. $(x-1)(x+2) = 4$, **so** $x^2 + x - 6 = 0$; **valid solution** $x = 2$.
2. **Domain** $x > 4$. $2x + 1 = 10(x-4)$, **so** $x = \frac{41}{8} = 5.125$.
3. **Domain** $x > 1$. $\log_2\left(\frac{x+5}{x-1}\right) = 2$, **so** $x = 3$.
4. **Need** $12 - x^2 > 0$; **domain** $-2\sqrt{3} < x < 2\sqrt{3}$.

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H. EXPONENTIAL EQUATIONS USING LOGARITHMS

1. $x+1=\log_3 7$, **so** $x=\frac{\log 7}{\log 3}-1\approx 0.771$.
2. $500(1.07)^t=1000$ **gives** $(1.07)^t=2$; $t=\frac{\log 2}{\log 1.07}\approx 10.24$ **years.**
3. $120(0.82)^t=30$ **gives** $(0.82)^t=0.25$; $t=\frac{\log(0.25)}{\log(0.82)}\approx 6.98$ **hours.**

I. COMMUNICATING WITH LOGS

1. $\log_3\left(\frac{p^2}{\sqrt{q}}\right)=2\log_3 p-\frac{1}{2}\log_3 q=2\alpha-\frac{1}{2}\beta$.
2. $b^3=64$, **so** $b=4$.

J. EXTENDED IB-STYLE APPLICATIONS

1. $250=10^d$; $d=\log_{10}(250)\approx 2.40$.
2. $1200=800(1.048)^t$; $t=\frac{\log(1.5)}{\log(1.048)}\approx 8.64$ **years.**
3. $10\log_{10}(8)\approx 9.0$ **dB.**

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K. NON-ROUTINE REASONING

1. **Domain** $x > 1$. $(x-1)(x+3) = 5x+3$; $x^2 - 3x - 6 = 0$. **Valid solution** $x = \frac{3+\sqrt{33}}{2}$.
2. **Substitute** $x = 4$: $\log_2(4) + \log_2(4-k) = 3$, **so** $4(4-k) = 8$; **hence** $k = 2$.
3. **Let** $\log_a x = \log_a y = c$. **Then** $x = a^c$ **and** $y = a^c$, **so** $x = y$.

L. REFLECTION

1. **Combining logarithms and exponentiating can create candidates that fail the original domain restrictions.**
2. **Many answers are possible, for example** $\log_2(x-1) = 3$ **has the single solution** $x = 9$.

LESSON FORMULA SUMMARY

$$a^{n/m} = \sqrt[m]{a^n} = (\sqrt[m]{a})^n$$

$$\log_a(xy) = \log_a x + \log_a y$$

$$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$$

$$\log_a(x^m) = m\log_a x$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$